1972

Sensitivity analysis of state estimation for power systems

Thomas Andrew Stuart
Iowa State University

STUART, Thomas Andrew, 1941-
SENSITIVITY ANALYSIS OF STATE ESTIMATION FOR POWER SYSTEMS.

Iowa State University, Ph.D., 1972
Engineering, electrical

Sensitivity analysis of state estimation for power systems

by

Thomas Andrew Stuart

A Dissertation Submitted to the

Graduate Faculty in Partial Fulfillment of

The Requirements for the Degree of

DOCTOR OF PHILOSOPHY

Major: Electrical Engineering

Approved:

Signature was redacted for privacy.

In Charge of Major Work

Signature was redacted for privacy.

For the Major Department

Signature was redacted for privacy.

For the Graduate College

Iowa State University
Ames, Iowa

1972

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

I. INTRODUCTION

This dissertation presents the results of a study of how various modeling errors affect the statistical estimate of the static state of an electrical power system. Criteria for evaluating the sensitivity of the estimates are developed, and the method is applied to a simulated model of an actual transmission system. The study discusses the mathematical aspects of the sensitivity analysis and considers many of the practical problems involved.

As the size and complexity of EHV transmission systems have increased over the past few years, electric utilities have experienced a need to increase the number of on-line electrical measurements to be used for monitoring and control purposes. One natural approach to this problem would be to measure every variable of interest; however, this is not only expensive but unnecessary since many variables can be calculated from others using a digital computer on an on-line basis. While there are many variables that provide useful information about a transmission system, two of particular interest are the voltage magnitude and phase angle at each bus. First of all, these are useful quantities in themselves since voltage magnitudes must be maintained at certain levels, and the phase angle separation between different buses gives a good indication of when the system is approaching a marginally stable operating condition. In addition to this, these variables can be used to calculate directly a wide range of additional information about the system, such as injected power at the buses, line flow power, and line currents.

It should also be emphasized that on-line calculations are presently the only practical means of determining phase angles since direct measurement of these variables require very sophisticated instruments.

Therefore, in a static sense, the bus voltage magnitudes and phase angles can be said to describe the behavior or present "state" of the system. Although this may be an unfortunate choice of terms, it has become standard practice to refer to these variables as the state variables of the system. The reason that this may be misleading is that in systems theory the state of a system is defined to be a set of variables which, along with a mathematical model and the inputs to the system, are adequate for predicting the dynamic behavior of the system. The variables chosen here do not necessarily meet this requirement, since accurate dynamic models have not even been developed, and the time behavior of such variables as frequency and the characteristics of generators have been completely ignored. Bearing this in mind however we can proceed, remembering that the term "state" in this context refers only to the static condition of the system.

The task to be performed then, is to take certain system measurements such as bus and line power levels and bus voltage magnitudes and calculate all of the bus voltage magnitudes and phase angles, or state, of the system. With the exception of voltage magnitudes, all measured quantities will be nonlinear functions of the state variables so iterative techniques resembling load flow methods must be used.

Many electrical power systems presently have large numbers of on-line measurements available, but it is quite likely that some additional ones will be needed to determine the state. There is also a strong possibility that some of the present measurements will be redundant for state calculation purposes, so that once the necessary additions have been made there may be more measurements than state variables. It is also quite likely that we may wish to add some redundant measurements so that a solution can still be obtained even if certain data is lost in processing.

Although a solution can be found without them, these redundant measurements do contain useful information about the system, and they can be combined with the other measurements to produce a more accurate result. Statistical estimation techniques can be utilized to determine how the measurements should be combined, hence the process is referred to as state estimation. The increased accuracy of such a process follows from the fact that the resulting estimate will have a lower variance than if the redundant measurements are ignored, and there is a tendency to alleviate the errors caused by bad data points.

An extremely important step in the application of estimation techniques to physical systems is to determine how errors in the model of the system will affect the accuracy of the resulting estimates. A sensitivity study of this sort will show the analyst which parameters must be known accurately and which ones are less critical. Once the relative effects of each parameter are known one can determine which ones should be known with greater precision to achieve the desired accuracy of the estimate.

This disertation is primarily concerned with how weighted least-squares estimates are affected by errors in such parameters as line resistance, inductance, and capacitance and unknown transformer tap ratios. The assumed statistical variances of the measurement errors are used in assigning a weight to each of the combined measurements, so errors in these quantities are also considered.

If the model of the system is correct and the measurement errors are normally distributed, the weighted least-squares estimates will be unbiased and will have the minimum variance among all unbiased estimates. For this reason, the criteria for evaluating the effects of modeling errors will be based on how the expected error and the variance of the resulting estimates will differ from the optimum values. This analysis method is also a necessary step in determining how errors in the estimates will affect the applications in which they are to be used. For example, if the state estimates are to be used for calculating unmeasured power levels, the sensitivity of each power level with respect to the state can be evaluated only when errors in the state itself have been determined. A few examples are included in the experimental results to demonstrate how these errors in the state estimates can affect the calculation of certain unmeasured power levels.

After the mathematical basis for the sensitivity analysis has been developed, the method will be applied to the model of an actual physical system. The model chosen for this simulation is based on the Iowa Power and Light Company's Central Division which is located in the vicinity of Des Moines, Iowa. This particular model consists of 58 buses and

69 transmission lines and involves bus voltages ranging from 46 to 345 Kv. An extensive amount of computer programming is required to perform this experiment, so the more important algorithms will be discussed along with appropriate flow charts.

II. REVIEW OF PROPOSED ESTIMATION TECHNIQUES

Several papers on state estimation of power systems have appeared recently, and most of the present work falls into one of the following categories:

1. Nonstatistical approach using weighted least-squares (references 24, 26, and 27)
2. Limiting the number of measurements to obtain a set of independent equations (references 9 and 26)
3. Kalman filtering approach (references 2, 3, and 6)
4. Statistical approach using weighted least-squares estimates (references 7, 10, 11 , 13, 14, 19, 20, 21, 22, 23, and 30)

As was noted in the introduction, the sensitivity analysis to be described will be limited to the statistical weighted least-squares approach. At first this may appear to be rather arbitrary, but there are a number of reasons for taking this attitude.

A nonstatistical weighted least-squares approach has some merit since it utilizes all of the available measurements, but the properties of the resulting estimate are not well defined in a mathematical sense. It has been suggested (references 24 and 26) that the measurements should be weighted according to which ones are the most "accurate" or "important", which seems reasonable intuitively, but just exactly what this means is still open to conjecture.

Limiting the analysis to a set of independent equations ignores all of the redundant measurements and, of course, all the information contained therein. Also, if a measurement is lost in processing there will be an infinite number of solutions, which amounts to no solution at all. However, it seems reasonable that this problem could be overcome by holding extra measurements in reserve to be used if necessary. This approach does have a potential computational advantage, since fewer calculations are required which reduces memory requirements and processing time.

It is interesting to note that the combined research group at Systems Control Incorporated and the Bonneville Power Administration, which is responsible for references 2, 3, 6, 10, 11, used a statistical weighted least-squares approach in their initial studies but then cast this aside in favor of the Kalman filter. Reference 2 indicates that computational advantages were the reason for doing this. This seems to be a very questionable decision however, since the resulting algorithm depends on some rather gross simplifications to the Kalman filter. In addition, the mathematical model required for this technique is basically incompatible with an electric power transmission system. The Kalman filter requires that the measurement errors be modeled as white noise, i.e., measurement errors that are completely uncorrelated from one time interval to another; but, the measurement errors in a power system appear to be predominately random bias errors (reference 5)[1] which do not vary (or at least are very slowly varying) with time. Unless the state vector to be estimated is augmented to include the bias errors there is no reason to expect the Kalman filter to yield results that will be optimum in any sense. Augmentation of the state

[1]W. H. Litzenberger, Bonneville Power Administration, Portland Oregon. Private communication to T. A. Stuart. July 6, 1971.

vector does not appear to be a very wise approach in this case however, since it would vastly increase the dimension of a system which may already contain several hundred states. Also, since the Kalman filter averages the last estimate with the present measurement, the time behavior of the system must be modeled. Models of this type for power systems are at present very crude and at best full of uncertainties (which is also pointed out in reference 2), but even if accurate models were available one is still faced with the bias error problem. Actually, it is possible to account for bias errors without augmentation of the state vector, and the development of such an approach is shown in Appendix A. The resulting algorithm is quite lengthy however, and it would undoubtedly require several approximations to ever be practical as an on-line estimator.

An examination of the statistical weighted least-squares approach, however, reveals many factors that make it very compatible with the power systems problem. Since measurement bias errors (or other types of errors for that matter) can arise from several independent sources, it is probably a reasonable approximation to assume that the total measurement error is normally distributed (by the central limit theorem of statistics). This being the case, when the weighted least-squares technique is applied to the linearized model it can be shown that the resulting estimate is unbiased and has the minimum variance among all unbiased estimates. This technique places no restrictions on the time behavior of the measurement errors, so it should give the same results for all types of measurement noise. Also, since each estimate is based only on measurements taken in the same time interval (which are assumed to be simultaneous), it is unnecessary to model the dynamics of the system.

In summary, it appears that statistical weighted least-squares is the only one of the above approaches that makes use of all the available information, is compatible with the physical system, and yields well defined results. Therefore it was decided to concentrate the sensitivity analysis on this technique and derive as much information about it as possible.

In spite of all the recent interest in this field, very little attention has been paid to the sensitivity of the estimator with respect to parameter errors. Reference 9, pages IV-17, 18, 26-30, includes a study of the sensitivity of line flow calculations with respect to random errors in transmission line parameters but omits such practical aspects as unknown transformer tap settings, errors in the measurement error covariance matrix, and the effect of redundant measurements. Reference 3 also discusses the sensitivity problem for the Kalman filter approach, but this study includes no data on how modeling errors affect the state estimates themselves.

III. WEIGHTED LEAST-SQUARES ESTIMATION

The estimator discussed in this section is basically the same as that of references 7, 10, 11, 13, 14, 19, 20, 21, 22, 23, and 30. The method of solution is slightly different however, and some of the mathematical properties are discussed in greater detail.

The state variables to be estimated are the voltage magnitudes and phase angles at each of the n buses of the system. Since phase angles are all relative to some fixed reference, the angle at the highest numbered bus is arbitrarily set equal to zero and all remaining angles are specified with respect to this. Therefore we are left to estimate 2n-1 state variables which will be denoted by the vector $\underline{x}$.

The available measurements will be represented by the m dimensional vector ($m \geq 2n-1$) of random variables, $\underline{Z}(k)$, where

$$\underline{Z}(k) = \underline{f}(\underline{x}(k)) + \underline{V}(k) \tag{3.1}$$

where, k = the time interval at which measurements are taken (all measurements within the same time interval are assumed to be simultaneous)

$\underline{f}(\underline{x}(k))$ = some nonlinear vector function of $\underline{x}(k)$ which is determined by the load flow equations of the system

$\underline{V}(k)$ = vector of the measurement errors where $\underline{V}(k)$ is assumed to be a vector of normally distributed random variables with mean = $\underline{0}$ and covariance matrix = R, which will be denoted by $\underline{V}(k) \sim N(\underline{0},R)$.

We now wish to take $\underline{Z}(k)$ and calculate some optimum estimate of $\underline{x}(k)$ which will be denoted by $\underline{X}(k)$.[1] First it must be decided what is meant by optimum. This could be quite an involved decision since there are many desirable and perhaps conflicting properties that certain different estimates possess. Of the various properties there are two in particular that we would like our estimate to have:

1. Unbiasedness

2. If possible, minimum variance among all unbiased estimates

Unbiasedness simply means that on the average the estimate will be equal to the quantity that is being estimated, or stated mathematically,

$$E(\underline{X}(k)/\underline{x}(k)) = \underline{x}(k) \tag{3.2}$$

where E denotes the expected value operator of statistics.

The minimum variance property simply indicates that the dispersion of the estimate about its expected value will be minimized, or that the diagonal terms of the following covariance matrix will be minimized,

$$\mathrm{cov.}(\underline{X}(k)) = [(\underline{X}(k) - E(\underline{X}(k)))(\underline{X}(k) - E(\underline{X}(k)))'] \tag{3.3}$$

where $'$ denotes the transpose of a vector or matrix.

To begin the search for an estimate which has the above properties, consider the $\underline{X}(k)$ which minimizes the following weighted squares cost function:

---

[1]Note that $\underline{x}(k)$ denotes a quantity to be estimated and $\underline{X}(k)$ is a random variable.

$$J(\underline{X}(k)) = \{[\underline{Z}(k) - \underline{f}(\underline{X}(k))]'R^{-1}[\underline{Z}(k) - \underline{f}(\underline{X}(k))]\} \tag{3.4}$$

Since $J(\underline{X}(k))$ is non-linear it is extremely difficult to obtain the required closed form solution for $\underline{X}(k)$ or to establish what mathematical properties it will possess. Therefore, an iterative approach is suggested.

Assuming that $\underline{f}(\underline{x}(k))$ is slowly varying with time and that $\underline{x}(k)$ is sufficiently close to some known $\underline{x}_o$, $\underline{f}(\underline{x}(k))$ can be linearized by approximating it equal to the first two terms of its Taylor series about $\underline{x}_o$,

$$\underline{f}(\underline{x}(k)) = \underline{f}(\underline{x}_o) + F(\underline{x}_o)(\underline{x}(k) - \underline{x}_o) \tag{3.5}$$

where $F(\underline{x}_o)$ is the Jacobian matrix of $\underline{f}(\underline{x}_o)$.

Substituting Equation 3.5 into Equation 3.4 leads to,

$$\begin{aligned} J(\underline{X}(k)) = \{&[\underline{Z}(k) - \underline{f}(\underline{x}_o) - F(\underline{x}_o)(\underline{X}(k) - \underline{x}_o)]'R^{-1} \\ &[\underline{Z}(k) - \underline{f}(\underline{x}_o) - F(\underline{x}_o)(\underline{X}(k) - \underline{x}_o)]\} \end{aligned} \tag{3.6}$$

It will presently be shown that the $\underline{X}(k)$ which minimizes Equation 3.6 can be found and has the above desired mathematical properties.

Note that the $\underline{X}(k)$ found by minimizing Equation 3.6 is only an approximation to the value which actually minimizes Equation 3.4, so the properties of the two will not necessarily be the same. The following approach will be taken in an attempt to resolve this discrepancy. Equations 3.5 and 3.6 can be used as the basis of a

Newton-Raphson iterative technique for finding the $\underline{X}(k)$ which minimizes Equation 3.4, where $\underline{x}_o$ is the value of $\underline{X}(k-1)$ found on the previous iteration. After each iteration, the $\underline{X}(k)$ found by minimizing Equation 3.6 will be substituted into Equation 3.4 and the result will be compared with the value of Equation 3.4 found from the previous iteration. When the difference between two iterations is less than some pre-determined value it will be assumed that $\underline{X}(k)$ is sufficiently close to the desired value that minimizes Equation 3.4.[1] This indicates that Equations 3.5 and 3.6 are also very accurate approximations at this point. Therefore we will assume that the system can be described by Equation 3.5 on this last iteration and we can investigate the properties of the estimate based on this model of the system.

A. Existence

Referring to Equation 3.5 and remembering that $\underline{V}(k) \sim N(\underline{0},R)$ it follows that

$$\underline{Z}(k) - \underline{f}(\underline{x}_o) + F(\underline{x}_o)\underline{x}_o \sim N(F(\underline{x}_o)\underline{x}(k),\ R) \tag{3.7}$$

Let

$$\underline{Y}(k) = R^{-\frac{1}{2}}(\underline{Z}(k) - \underline{f}(\underline{x}_o) + F(\underline{x}_o)\underline{x}_o) \tag{3.8}$$

$$U(\underline{x}_o) = R^{-\frac{1}{2}}F(\underline{x}_o) \tag{3.9}$$

---

[1] As pointed out in reference 20, it has not been proven that this iterative technique will necessarily converge, and justification for its use is based strictly on experimental evidence of satisfactory results.

therefore,

$$\underline{Y}(k) \sim N(U(\underline{x}_o)\underline{x}(k),\ I), \tag{3.10}$$

where I is the identity matrix.

We now wish to investigate the properties of the estimate $\underline{X}(k)$ which minimizes the linearized weighted squares cost function,

$$J(\underline{X}(k)) = \{[\underline{Z}(k) - \underline{f}(\underline{x}_o) - F(\underline{x}_o)(\underline{X}(k) - \underline{x}_o)]'R^{-1} \cdot [\underline{Z}(k) - \underline{f}(\underline{x}_o) - F(\underline{x}_o)(\underline{X}(k) - \underline{x}_o)]\} \tag{3.11}$$

Equation 3.11 is equivalent to,

$$J(\underline{X}(k)) = [(\underline{Y}(k) - U(\underline{x}_o)\underline{X}(k))'(\underline{Y}(k) - U(\underline{x}_o)\underline{X}(k))] \tag{3.12}$$

Let $\hat{\underline{X}}(k)$ denote the estimate which minimizes Equation 3.12. Naturally, any $\hat{\underline{X}}(k)$ which meets this requirement must also satisfy the following equation,[1]

$$\frac{\partial J(\hat{\underline{X}}(k))}{\partial \hat{\underline{X}}(k)} = -(\underline{Y}(k) - U(\underline{x}_o)\hat{\underline{X}}(k))'(I + I)U(\underline{x}_o) = \underline{0}' \tag{3.13}$$

therefore,

$$\hat{\underline{X}}'(k)U'(\underline{x}_o)U(\underline{x}_o) - \underline{Y}'(k)U(\underline{x}_o) = \underline{0}'$$

and

$$U'(\underline{x}_o)U(\underline{x}_o)\hat{\underline{X}}(k) - U'(\underline{x}_o)\underline{Y}(k) = \underline{0} \tag{3.14}$$

We can now investigate to determine if a solution for Equation 3.14 exists.

---

[1] (29, p. 94).

The rank of a matrix is defined to be the dimension of the vector space generated by the columns or rows of the matrix. The vector space generated by the columns of U will be denoted by M(U).

<u>Theorem 1</u>:[1] $M(U) = M(UU')$. That is, the vector spaces generated by the columns of U and UU' are the same. Hence, dimension of $M(U)$ = dimension of $M(UU')$ = rank of U = rank of UU'.

<u>Proof</u>: If $\underline{a}$ is a column vector such that $\underline{a}'U = \underline{0}' \Rightarrow \underline{a}'UU' = \underline{0}'$. Conversely, $\underline{a}'UU' = \underline{0}' \Rightarrow \underline{a}'UU'\underline{a} = 0 \Rightarrow \underline{a}'U = \underline{0}'$. Hence every vector orthogonal to U is also orthogonal to UU'. Therefore $M(U) = M(UU')$.

Now, $U'(\underline{x}_o)\underline{Y}(k) \in M(U'(\underline{x}_o))$, therefore $U'(\underline{x}_o)\underline{Y}(k) \in M(U'(\underline{x}_o)U(\underline{x}_o))$, and there exists some $\hat{\underline{X}}(k)$ such that $U'(\underline{x}_o)\underline{Y}(k) = U'(\underline{x}_o)U(\underline{x}_o)\hat{\underline{X}}(k)$.

We will now determine if the minimum of Equation 3.12 is unique. Let $\hat{\underline{X}}(k)$ be any solution to Equation 3.14 (which is not necessarily a unique solution).

$$\begin{aligned}(\underline{Y} - U\underline{X})'(\underline{Y} - U\underline{X}) &= (\underline{Y} - U\hat{\underline{X}} + U(\hat{\underline{X}} - \underline{X}))'(\underline{Y} - U\hat{\underline{X}} + U(\hat{\underline{X}} - \underline{X})) \\ &= (\underline{Y} - U\hat{\underline{X}})'(\underline{Y} - U\hat{\underline{X}}) + (\hat{\underline{X}} - \underline{X})'U'U(\hat{\underline{X}} - \underline{X}) \\ &\quad + \underline{Y}'U\hat{\underline{X}} - \underline{Y}'U\underline{X} - \hat{\underline{X}}'U'U\hat{\underline{X}} + \hat{\underline{X}}'U'U\underline{X} + \hat{\underline{X}}'U'\underline{Y} - \hat{\underline{X}}'U'U\hat{\underline{X}} \\ &\quad - \underline{X}'U'\underline{Y} + \underline{X}'U'U\hat{\underline{X}} \end{aligned} \tag{3.15}$$

[1](18, p. 27).

The last eight terms of Equation 3.15 sum to zero when we substitute $U'U\hat{\underline{X}} = U'\underline{Y}$. Therefore,

$$(\underline{Y} - U\underline{X})'(\underline{Y} - U\underline{X}) = (\underline{Y} - U\hat{\underline{X}})'(\underline{Y} - U\hat{\underline{X}}) + (\hat{\underline{X}} - \underline{X})'U'U(\hat{\underline{X}} - \underline{X})$$

$$\geq (\underline{Y} - U\hat{\underline{X}})'(\underline{Y} - U\hat{\underline{X}}) \qquad (3.16)$$

Equation 3.16 shows that when $\underline{X} = \hat{\underline{X}}$, $(\underline{Y} - U\hat{\underline{X}})'(\underline{Y} - U\hat{\underline{X}})$ is the unique minimum of $(\underline{Y} - U\underline{X})'(\underline{Y} - U\underline{X})$. We have not established the uniqueness of $\hat{\underline{X}}$ at this point, so it may still be any solution of Equation 3.14.

B. Uniqueness

Having established that a solution, $\hat{\underline{X}}(k)$, exists for Equation 3.14 and that it provides a unique minimum, we can investigate the method of finding such a solution and try to determine if it will be unique. If $(U'(\underline{x}_o)U(\underline{x}_o))^{-1}$ exists the solution is obvious, but the question remains as to whether this is a reasonable assumption.

From Theorem 1, it follows that if the rank of $U'(\underline{x}_o) = 2n-1$, the number of unknown state variables, then the rank of $U'(\underline{x}_o)U(\underline{x}_o) = 2n-1$. However, $U'(\underline{x}_o)U(\underline{x}_o)$ is a square matrix with dimension $= 2n-1$, and therefore since rank = dimension it follows that $(U'(\underline{x}_o)U(\underline{x}_o))^{-1}$ exists.[1] Furthermore, since the inverse of a matrix is unique[2], it follows that,

---

[1](17, p. 60).

[2](17, p. 41).

$$\underline{\hat{X}}(k) = (U'(\underline{x}_o)U(\underline{x}_o))^{-1}U'(\underline{x}_o)\underline{Y}(k) \tag{3.17}$$

will be the unique solution to Equation 3.14.

The derivation of Equation 3.17, of course, depends on the assumption that the rank of $U(\underline{x}_o)$ is equal to 2n-1. It is also possible to relate this assumption to the Jacobian matrix, $F(\underline{x}_o)$, which may provide more insight to the physical problem. $R^{-\frac{1}{2}}$ in Equation 3.9 is nonsingular, so it follows that $U(\underline{x}_o)$ is equivalent to $F(\underline{x}_o)$ and they have the same rank.[1] In other words, it is equivalent to assume that the rank of $F(\underline{x}_o)$ is 2n-1 (recall that $F(\underline{x}_o)$ has 2n-1 columns and at least 2n-1 rows). There is no known guarantee that this will always be the case for power systems, but this assumption will be made so that the existence of $(U'(\underline{x}_o)U(\underline{x}_o))^{-1}$ can also be assumed.

Equation 3.17 can be rewritten directly in terms of the system parameters as,

$$\underline{\hat{X}}(k) = (F'(\underline{x}_o)R^{-1}F(\underline{x}_o))^{-1}F'(\underline{x}_o)R^{-1}(\underline{Z}(k) - \underline{f}(\underline{x}_o)) + \underline{x}_o \tag{3.18}$$

where $\underline{\hat{X}}(k)$ is known to be the unique estimate that locates the unique minimum of Equation 3.11.

---

[1](17, p. 61).

C. Unbiasedness

Substituting Equations 3.1 and 3.5 into Equation 3.18 and taking the expected value yields,

$$E(\underline{\hat{X}}(k)) = (F'(\underline{x}_o)R^{-1}F(\underline{x}_o))^{-1}F'(\underline{x}_o)R^{-1}[F(\underline{x}_o)\underline{x}(k) + E(\underline{V}(k))]$$

$$= \underline{x}(k) \tag{3.19}$$

or equivalently,

$$E(\underline{\hat{X}}(k)/\underline{x}(k)) = \underline{x}(k) \tag{3.20}$$

meaning that $\underline{\hat{X}}(k)$ is unbiased.

D. Minimum Variance Among All Unbiased Estimates

Let Kg denote the class of all unbiased estimators (functions of $\underline{Y}$) of g, a specified scalar valued function of $\underline{x}$, and let K_o be the class of all scalar valued functions of $\underline{Y}$ having zero expectation. Thus $T \in Kg$ iff $E(T/\underline{x}) = g(\underline{x})$ for each $\underline{x}$, and $S \in Ko$ iff $E(S/\underline{x}) = 0$ for each $\underline{x}$.

<u>Theorem 2</u>:[1] The necessary and sufficient condition that an estimator $T \in Kg$ has minimum variance at the value $\underline{x} = \underline{x}_1$ is that $\mathrm{cov}(T,S/\underline{x}_1) = 0$ for every $S \in K_o$, such that $\mathrm{var}(S/\underline{x}_1) < \infty$ provided $\mathrm{var}(T/\underline{x}_1) < \infty$.

<u>Proof</u>: To simplify the following expressions, use the notation $\mathrm{var}(\cdot) \equiv \mathrm{var}(\cdot/\underline{x}_1)$ and $\mathrm{cov}(\cdot) \equiv \mathrm{cov}(\cdot/\underline{x}_1)$. The necessity is proved by considering $(T+\lambda S) \in Kg$ for arbitrary λ and showing that, for any λ in the interval $(0, -2\mathrm{cov}(T,S)/\mathrm{var}(S))$,

$\mathrm{var}(T+\lambda S) = [\mathrm{var}(T) + 2\lambda \mathrm{cov}(T,S) + \lambda^2 \mathrm{var}(S)] \leq \mathrm{var}(T)$ unless $\mathrm{cov}(T,S) = 0$, i.e., assume $\lambda = -(2b)\mathrm{cov}(T,S)/\mathrm{var}(S)$ for $0 < b < 1$,

[1](18, pp. 257, 258).

therefore,

$$\text{var}(T+\lambda S) = \text{var}(T) - \frac{(4b)\text{cov}^2(T,S)}{\text{var}(S)} + \frac{(4b^2)\text{cov}^2(T,S)}{\text{var}(S)}$$

$$= \text{var}(T) - \frac{(1-b)(4b)\text{cov}^2(T,S)}{\text{var}(S)} \leq \text{var}(T)$$

unless $\text{cov}(T,S) = 0$.

To prove sufficiency, let $T^* \in Kg$ be another estimator such that $\text{var}(T^*) < \infty$ at $\underline{x}_1$. Then $(T - T^*) \in Ko$, and by the condition $\text{cov}(T,S/\underline{x}_1) = 0$, $\text{cov}(T(T - T^*)/x_1) = 0$ or $\text{var}(T) = \text{cov}(T,T^*)$, and $(\text{var}(T))^{\frac{1}{2}} = \rho(\text{var}(T^*))^{\frac{1}{2}}$ $\leq (\text{var}(T^*))^{\frac{1}{2}}$ where ρ is the correlation between T and T^*, $(\rho \leq 1)$.

Returning now to the original system of equations, we have from Equation 3.10,

$$\underline{Y} \sim N(U\underline{x},I)$$

For every $c(\underline{Y}) \in Ko$ we have

$$\int\cdots\int \; c(\underline{y})\exp-(\underline{y} - U\underline{x})'(\underline{y} - U\underline{x})/2 \; dy_1 \cdots\cdots dy_m = 0$$

Differentiating the above integral w.r.t. $\underline{x}$,

$$\int\cdots\int c(\underline{y})(\underline{y} - U\underline{x})'U\exp - (\underline{y} - U\underline{x})'(\underline{y} - U\underline{x})/2 \; dy_1 \cdots dy_m = \underline{0}'$$

$$\int\cdots\int c(\underline{y})(U'\underline{y} - U'U\underline{x})\exp - (\underline{y} - U\underline{x})'(\underline{y} - U\underline{x})/2dy_1\cdots dy_m = \underline{0}$$

and since $(U'U)^{-1}$ is assumed to exist,

$$\int\cdots\int c(\underline{y})[(U'U)^{-1}U'\underline{y} - \underline{x}] \exp - (\underline{y}-U\underline{x})'(\underline{y}-U\underline{x})/2 dy_1 \cdots dy_m = \underline{0}$$

or for each $\hat{X}_i$, $\text{cov}(\hat{X}_i, c(\underline{Y})) = 0$

$\hat{\underline{X}} = (U'U)^{-1}U'\underline{Y}$ is an unbiased estimate of $\underline{x}$ so by applying Theorem 2 to each $\hat{X}_i$ it follows that each $\hat{X}_i$ has the minimum variance of all such estimates.

In summary it can be concluded that the $\hat{\underline{X}}$ which minimizes the weighted squares cost function of Equation 3.11 provides an estimate which

1. Exists (i.e., a solution can be found)
2. Is unique and provides the unique minimum value of the cost function
3. Is unbiased
4. Each $\hat{X}_i$ has the minimum variance among all unbiased estimates

IV. COVARIANCE CALCULATIONS

A. Covariance of the Optimum Estimate

From Equation 3.10, $\underline{Y} \sim N(U\underline{x}, I)$, and from Equation 3.17, $\hat{\underline{X}} = (U'U)^{-1}U'\underline{Y}$. Therefore

$$P(k) \equiv \mathrm{cov}(\hat{\underline{X}}) = (U'U)^{-1}U'[(U'U)^{-1}U']' = (U'U)^{-1}$$

and from Equation 3.9,

$$P(k) = (F'(\underline{x}_o)R^{-1}F(\underline{x}_o))^{-1} \qquad (4.1)$$

B. Calculated and Actual Covariance of the Estimate When Modeling Errors are Present

This section is concerned with the calculated and actual covariances that result when any combination of the following modeling errors are present:

1. Incorrect $\underline{f}_c$ and F_c instead of correct $\underline{f}$ and F
2. Incorrect R_c instead of correct R

When unknown modeling errors are present, the calculation procedure of course remains the same, so that the calculated estimate, $\hat{\underline{X}}_c$, and covariance matrix, P_c, can be represented by

$$\hat{\underline{X}}_c = (F_c'R_c^{-1}F_c)^{-1}F_c'R_c^{-1}(\underline{Z} - \underline{f}_c) + \underline{x}_{oc} \qquad (4.2)$$

$$P_c = (F_c'R_c^{-1}F_c)^{-1} \qquad (4.3)$$

The covariance of $\underline{Z}$ is R, so from Equation 4.2 we can also find the actual covariance, P_a,

$$P_a = \mathrm{cov}(\hat{\underline{X}}_c) = (F_c' R_c^{-1} F_c)^{-1} F_c' R_c^{-1} R R_c^{-1} F_c (F_c' R_c^{-1} F_c)^{-1} \tag{4.4}$$

or

$$P_a = P_c F_c' R_c^{-1} R R_c^{-1} F_c P_c \tag{4.5}$$

It is interesting to note the following:

1. If $R_c = R$, Equation 4.5 shows that $P_a = P_c$ regardless of any errors in $\underline{f}_c$ and F_c. This is exactly what we should expect since the estimate $\hat{\underline{X}}_c$ in Equation 4.2 is a known linear function of the random variable, $\underline{Z}$. Therefore if the covariance, R, of $\underline{Z}$ is known, the actual covariance, P_a, of $\hat{\underline{X}}_c$ can be found.
2. If $F_c = F$ and F^{-1} exists, $\hat{\underline{X}}_c$ from Equation 4.2 can be written,

$$\begin{aligned} \hat{\underline{X}}_c &= F^{-1} R_c (F')^{-1} F' R_c^{-1} (\underline{Z} - \underline{f}_c) + \underline{x}_{oc} \\ &= F^{-1} (\underline{Z} - \underline{f}_c) + \underline{x}_{oc} \end{aligned} \tag{4.6}$$

Therefore, $\hat{\underline{X}}_c$ is no longer a function of R_c, i.e., we are no longer assigning relative weights to the measurements since F is square and we have the same number of measurements as we have state variables. In this case we have,

$$P_a = F^{-1} R (F^{-1})' = (F' R^{-1} F)^{-1} = P \tag{4.7}$$

regardless of any errors in R_c .

C. Discussion

It seems appropriate at this point to summarize the results of the covariance calculations of the previous sections. The covariance of the optimum estimate is,

$$P(k) = (F'(\underline{x}_o)R^{-1}F(\underline{x}_o))^{-1} \tag{4.1}$$

The calculated covariance when modeling errors are present is,

$$P_c(k) = (F_c'(\underline{x}_{oc})R_c^{-1}F_c(\underline{x}_{oc}))^{-1} . \tag{4.3}$$

The actual covariance of the calculated estimate when modeling errors are present is,

$$P_a(k) = P_c(k)F_c'(\underline{x}_{oc})R_c^{-1}RR_c^{-1}F_c(\underline{x}_{oc})P_c(k) . \tag{4.5}$$

V. MAGNITUDE OF THE EXPECTED ERROR

The purpose here is to determine the expected or the average error that will result in the estimate when modeling errors are present. To determine this we shall assume that we are given the actual value of the state vector, $\underline{x}$, and then this can be compared with the expected value of the estimate, $E(\underline{\hat{X}}_c)$.

For the optimum estimate with no modeling errors present,

$$E[(\underline{\hat{X}} - \underline{x})/\underline{x}] = \underline{0} \tag{5.1}$$

since $\underline{\hat{X}}$ is unbiased.

From Equation 4.2,

$$\underline{\hat{X}}_c = (F_c' R_c^{-1} F_c)^{-1} F_c' R_c^{-1} (\underline{Z} - \underline{f}_c) + \underline{x}_{oc} \tag{5.2}$$

and

$$\underline{Z} = \underline{f} + F(\underline{x} - \underline{x}_o) + \underline{V} \tag{5.3}$$

Therefore,

$$E[(\underline{\hat{X}}_c - \underline{x})/\underline{x}] = (F_c' R_c^{-1} F_c)^{-1} F_c' R_c^{-1} (\underline{f} - \underline{f}_c - F\underline{x}_o + F\underline{x}) + \underline{x}_{oc} - \underline{x} \tag{5.4}$$

Naturally,

$$|E[(\underline{\hat{X}}_c - \underline{x})/\underline{x}]| \geq |E[(\underline{\hat{X}} - \underline{x})/\underline{x}]| \tag{5.5}$$

VI. CRITERIA FOR EVALUATING ESTIMATE ERRORS

Estimation techniques provide results that are optimum only on an average basis, so we are naturally interested in what the average error will be and how the individual errors will be distributed about this average. In this study the estimates have been chosen to provide the minimum variance among all unbiased estimates, so it is logical to use these two indices for evaluating the effects of modeling errors. The expected value of the estimate error provides a measure of the magnitude of the average error, and the variance of the estimate provides a measure of how it will be dispersed about its average value. Therefore both of these criteria should be examined when evaluating the accuracy of an estimate. The use of either one without the other can produce misleading results, as can be demonstrated by the following examples.

<u>Example 1</u>: Scalar case. Let $R = 1$, $F = 1$, $R_c = 2$, $F_c = 2$.
Therefore from Equations 4.1, 4.3, and 4.4, $P = 1$, $P_c = \frac{1}{2}$, $P_a = \frac{1}{4}$.
The results indicate that the actual and calculated variances are less than the variance of the optimum estimate. The reason for this is that the calculated estimate, $\hat{\underline{X}}_c$, is no longer unbiased because modeling errors are present. $\hat{\underline{X}}$ produces the minimum variance only in the class of all unbiased estimates, and there may be any number of biased estimates that have a lower variance. Therefore comparing the diagonal terms of P, P_c, and P_a will be rather meaningless without considering the expected values of the estimates themselves.

Example 2: Scalar case with two measurements.

$$F_c = F = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \quad \underline{f} \simeq \underline{f}_c, \quad x_o \simeq x_{oc}, \quad R_c^{-1} = \begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix}, \quad R^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix}$$

Equation 5.4 indicates that for this case $E[(\hat{\underline{X}}_c - \underline{x})/\underline{x}] \simeq \underline{0}$ or that on the average, $\hat{\underline{X}}_c$ will be unbiased, regardless of the errors in R_c. However we have from Equations 4.1 and 4.4, $P = 0.059$, $P_a = 0.140$, indicating that the actual variance of this estimate will be considerably greater than if R_c were correct.

The purpose of the above examples is to demonstrate that the expected error and the variance of the estimate both contain important information about the estimate and that each is incomplete without the other. The results show that:

1. It is possible to obtain estimates which will be grouped closer to their average than the optimum estimate will be, but these estimates may be biased, i.e., on the average they will not be equal to the $\underline{x}$ that we are trying to estimate.
2. A calculated estimate, $\hat{\underline{X}}_c$, may be unbiased and yet have a wider dispersion about its average value than the optimum estimate.

The following procedure will now be formulated for evaluating the effects of modeling errors. For convenience, the expected error and covariance equations for the optimum and calculated estimates are repeated below:

$$E[(\hat{\underline{X}} - \underline{x})/\underline{x}] = \underline{0} \tag{5.1}$$

$$E[(\hat{\underline{X}}_c - \underline{x})/\underline{x}] = (F_c'R_c^{-1}F_c)^{-1}F_c'R_c^{-1}[\underline{f} - \underline{f}_c + F(\underline{x} - \underline{x}_o)] - (\underline{x} - \underline{x}_{oc}) \tag{5.4}$$

$$P = (F'R^{-1}F)^{-1} \tag{4.1}$$

$$P_a = (F_c'R_c^{-1}F_c)^{-1}F_c'R_c^{-1}R\, R_c^{-1}F_c(F_c'R_c^{-1}F_c)^{-1} \tag{4.4}$$

If desired, various $\underline{x}(k)$ vectors can be selected to represent different system loading conditions and to study the effects of erroneous values of $\underline{f}_c$, F_c and R_c for each $\underline{x}(k)$. For each $\underline{x}(k)$ it will also be necessary to simulate a set of measurements $\underline{Z}(k)$ which contain random errors. Equations 4.1, 4.4, and 5.4 are dependent on $\underline{x}_o$ and $\underline{x}_{oc}$ which, in turn, are dependent on the measurements; therefore, these equations will be dependent on the measurement errors. The question then arises as to how these measurement errors should be chosen. One method would be to resort to a Monte Carlo approach where, 1) a large number of random errors are simulated, 2) Equations 4.1, 4.4, and 5.4 are found for each simulation, and 3) the results are then averaged. Such an approach requires many simulations and excessive amounts of computer time however, so a simplified technique is suggested. In this study we are not so much interested in how the estimate is affected by normal measurement errors as we are in how it is affected by errors in the model. Equations 4.1, 4.4, and 5.4 should be capable of demonstrating the effects of these modeling errors for each set of measurement errors that we simulate. Therefore, it seems reasonable to restrict ourselves to one test set of measurement errors and then study the relative effects of various modeling errors based on that test set. Using this approach, it is quite reasonable to limit the study to measurements which are noise free since $\underline{V} = \underline{0}$ is a perfectly valid choice of errors.

In summary then, the following approach will be used to evaluate the effects of modeling errors:

1. If desired, the system can be studied under various loading conditions, as represented by different values of $\underline{x}(k)$.
2. Noise free measurements will be used for the test case, and all results will be compared on this basis.
3. Equation 5.4 will be used to study the expected or average error of $\hat{\underline{X}}_c$.
4. The diagonal terms of Equation 4.4 will be used to study the actual dispersion of $\hat{\underline{X}}_c$ about its average value, and these will be compared with the dispersion of the optimum estimate, $\hat{\underline{X}}$, which is given by the diagonal terms of Equation 4.1.

As noted in Section I, this analysis method provides a necessary step in determining how errors in the estimates will affect the applications in which they are to be used. In the calculation of certain unmeasured quantities such as power levels, the sensitivity of each calculation with respect to the state can be evaluated only when the errors in the state itself have been determined. One method of evaluating the effects of errors in the state is simply to compare the calculations, 1) using the true state, and 2) using the true state + the expected error. A few examples are included in Section XII to demonstrate how these errors in the state estimates can affect the calculation of certain unmeasured line flow power levels.

VII. RELATION BETWEEN STATE VARIABLES AND MEASUREMENT EQUATIONS[1]

The state variable vector, $\underline{x}$, can be written as follows:

$$\underline{x}(k) = \begin{bmatrix} \underline{e}(k) \\ \\ \underline{\delta}(k) \end{bmatrix} \qquad (7.1)$$

where $\underline{x}(k)$ = (2n-1) dimensional vector

$\underline{e}(k)$ = n dimensional vector of bus voltage magnitudes

$\underline{\delta}(k)$ = n - 1 dimensional vector of bus voltage phase angles (the angle at the nth bus is arbitrarily set = 0^o).

The vector, $\underline{f}(\underline{x}(k))$, in Equation 3.1 may consist of the following quantities:

1. Real and reactive power injected at a bus
2. Real and reactive line flow power
3. Bus voltage magnitudes

These quantities were chosen because they are commonly measured. To simplify the computer program, it will be assumed that all power measurements will include the real and reactive components. The basis for this assumption is that if one is available, very little hardware is required to obtain the other.

Therefore we can write,

---

[1](25, Chapter 8).

$$\underline{f}(\underline{x}(k)) = \begin{bmatrix} \underline{p}(\underline{x}(k)) \\ \underline{q}(\underline{x}(k)) \\ \underline{g}(\underline{x}(k)) \\ \underline{h}(\underline{x}(k)) \\ \underline{e}(\underline{x}(k)) \end{bmatrix} \tag{7.2}$$

where $\underline{f}(\underline{x}(k))$ = vector of length $\geq$ 2n-1

$\underline{p}(\underline{x}(k))$ = real injected bus powers for which a measurement is available

$\underline{q}(\underline{x}(k))$ = reactive injected bus powers for which a measurement is available

$\underline{g}(\underline{x}(k))$ = real line flow powers for which a measurement is available

$\underline{h}(\underline{x}(k))$ = reactive line flow powers for which a measurement is available

$\underline{e}(\underline{x}(k))$ = bus voltage magnitudes for which a measurement is available

The lengths of the vectors $\underline{p}$, $\underline{q}$, $\underline{g}$, $\underline{h}$ and $\underline{e}$, will vary, depending on the measurement configuration.

A pi equivalent circuit will be used for each transmission line; and a typical configuration with line 1-4 open at one end is shown in Figure 7.1.

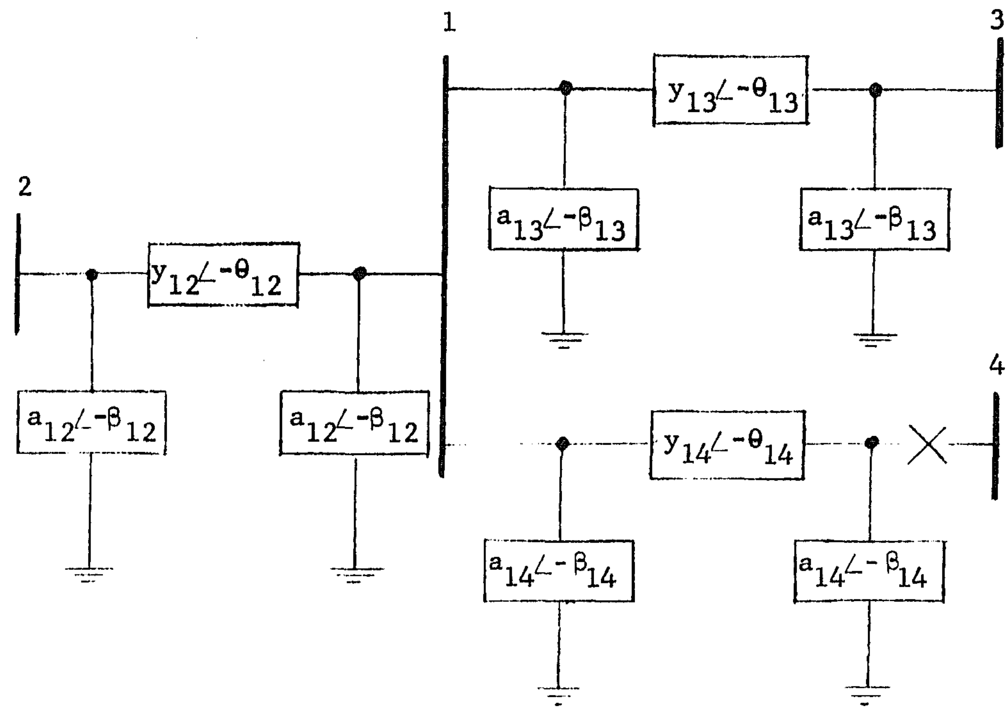


Figure 7.1. Typical line configuration

The transmission line parameters are defined as follows:

$\underline{y}$ = vector of magnitudes of series line admittances

$-\underline{\theta}$ = vector of phase angles of series line admittances

$\underline{t}$ = vector of the magnitudes of the total shunt admittances at each bus

$-\underline{\phi}$ = vector of the phase angles of the total shunt admittances at each bus

$\underline{a}$ = vector of the magnitudes of the shunt admittances at each end of a line

$-\underline{\beta}$ = vector of the phase angles of the shunt admittances at each end of a line

For Figure 7.1, it follows that,

$$p_1 + jq_1 = e_1\angle\delta_1[(e_2\angle\delta_2 - e_1\angle\delta_1)(y_{12}\angle{-\theta_{12}})$$
$$+ (e_3\angle\delta_3 - e_1\angle\delta_1)(y_{13}\angle{-\theta_{13}}) - (e_1\angle\delta_1)(t_1\angle{-\phi_1})]^* \tag{7.3}$$

$$p_1 + jq_1 = e_1 e_2 y_{12} \cos(\theta_{12} + \delta_1 - \delta_2) + e_1 e_3 y_{13} \cos(\theta_{13} + \delta_1 - \delta_3)$$
$$-e_1^2 [y_{12} \cos(\theta_{12}) + y_{13} \cos(\theta_{13}) + t_1 \cos(\phi_1)]$$
$$+je_1 e_2 y_{12} \sin(\theta_{12} + \delta_1 - \delta_2) + je_1 e_3 y_{13} \sin(\theta_{13} + \delta_1 - \delta_3)$$
$$-je_1^2 [y_{12} \sin(\theta_{12}) + y_{13} \sin(\theta_{13}) + t_1 \sin(\phi_1)] \tag{7.4}$$

From Equation 7.3 the following general result follows by inference,

$$p_i = e_i \sum_{\substack{j=1 \\ i\neq j}}^{n} e_j y_{ij} \cos(\theta_{ij} + \delta_i - \delta_j) - e_i^2 \sum_{\substack{j=1 \\ i\neq j}}^{n} y_{ij} \cos(\theta_{ij})$$
$$- e_i^2 t_i \cos(\phi_i) \tag{7.5}$$

$$q_i = e_i \sum_{\substack{j=1 \\ i\neq j}}^{n} e_j y_{ij} \sin(\theta_{ij} + \delta_i - \delta_j) - e_i^2 \sum_{\substack{j=1 \\ i\neq j}}^{n} y_{ij} \sin(\theta_{ij})$$
$$- e_i^2 t_i \sin(\phi_i) \tag{7.6}$$

Likewise for the line flow powers,

$$g_{ij} = e_i e_j y_{ij} \cos(\theta_{ij} + \delta_i - \delta_j) - e_i^2 y_{ij} \cos(\theta_{ij}) - e_i^2 a_{ij} \cos(\beta_{ij}) \tag{7.7}$$

$$h_{ij} = e_i e_j y_{ij} \sin(\theta_{ij} + \delta_i - \delta_j) - e_i^2 y_{ij} \sin(\theta_{ij}) - e_i^2 a_{ij} \sin(\beta_{ij}) \tag{7.8}$$

The terms of the Jacobian, F, can be written,

$$F = \left[\begin{array}{c:c} F_{11} & F_{12} \\ \hdashline F_{21} & F_{22} \\ \hdashline F_{31} & F_{32} \\ \hdashline F_{41} & F_{42} \\ \hdashline F_{51} & F_{52} \end{array}\right]$$

where for F_{11},

$$\frac{\partial p_i}{\partial e_i} = \sum_{\substack{j=1 \\ i \neq j}}^{n} e_j y_{ij} \cos(\theta_{ij} + \delta_i - \delta_j) - 2e_i \sum_{\substack{j=1 \\ i \neq j}}^{n} y_{ij} \cos(\theta_{ij})$$

$$- 2e_i t_i \cos(\phi_i) \tag{7.9}$$

$$\underset{i \neq j}{\frac{\partial p_i}{\partial e_j}} = e_i \sum_{\substack{j=1 \\ i \neq j}}^{n} y_{ij} \cos(\theta_{ij} + \delta_i - \delta_j) \tag{7.10}$$

for F_{12},

$$\frac{\partial p_i}{\partial \delta_i} = - e_i \sum_{\substack{j=1 \\ i \neq j}}^{n} e_j y_{ij} \sin(\theta_{ij} + \delta_i - \delta_j) \tag{7.11}$$

$$\left.\frac{\partial p_i}{\partial \delta_j}\right|_{i \neq j} = - \frac{\partial p_i}{\partial \delta_i} \tag{7.12}$$

for F_{21},

$$\frac{\partial q_i}{\partial e_i} = \sum_{\substack{j=1 \\ i \neq j}}^{n} e_i y_{ij} \sin(\theta_{ij} + \delta_i - \delta_j) - 2e_i \sum_{\substack{j=1 \\ i \neq j}}^{n} y_{ij} \sin(\theta_{ij})$$

$$- 2e_i t_i \sin(\phi_i) \tag{7.13}$$

$$\left.\frac{\partial q_i}{\partial e_j}\right|_{i \neq j} = e_i \sum_{\substack{j=1 \\ i \neq j}}^{n} y_{ij} \sin(\theta_{ij} + \delta_i - \delta_j) \tag{7.14}$$

for F_{22},

$$\frac{\partial q_i}{\partial \delta_i} = e_i \sum_{\substack{j=1 \\ i \neq j}}^{n} e_j y_{ij} \cos(\theta_{ij} + \delta_i - \delta_j) \tag{7.15}$$

$$\left.\frac{\partial q_i}{\partial \delta_j}\right|_{i \neq j} = - \frac{\partial q_i}{\partial \delta_i} \tag{7.16}$$

for F_{31},

$$\frac{\partial g_i}{\partial e_i} = e_j y_{ij} \cos(\theta_{ij} + \delta_i - \delta_j) - 2e_i y_{ij} \cos(\theta_{ij}) - 2e_i a_{ij} \cos(\beta_{ij}) \tag{7.17}$$

$$\frac{\partial g_i}{\partial e_j} = e_i y_{ij} \cos(\theta_{ij} + \delta_i - \delta_j) \tag{7.18}$$

$i \neq j$

for F_{32},

$$\frac{\partial g_i}{\partial \delta_i} = - e_i e_j y_{ij} \sin(\theta_{ij} + \delta_i - \delta_j) \tag{7.19}$$

$$\frac{\partial g_i}{\partial \delta_j} = - \frac{\partial g_i}{\partial \delta_i} \tag{7.20}$$

$i \neq j$

for F_{41},

$$\frac{\partial h_i}{\partial e_i} = e_j y_{ij} \sin(\theta_{ij} + \delta_i - \delta_j) - 2e_i y_{ij} \sin(\theta_{ij}) - 2e_i a_{ij} \sin(\beta_{ij}) \tag{7.21}$$

$$\frac{\partial h_i}{\partial e_j} = e_i y_{ij} \sin(\theta_{ij} + \delta_i - \delta_j) \tag{7.22}$$

$i \neq j$

for F_{42},

$$\frac{\partial h_i}{\partial \delta_i} = e_i e_j y_{ij} \cos(\theta_{ij} + \delta_i - \delta_j) \tag{7.23}$$

$$\frac{\partial h_i}{\partial \delta_j} = - \frac{\partial h_i}{\partial \delta_i} \quad (7.24)$$
$$i \neq j$$

for F_{51},

$$\frac{\partial e_i}{\partial e_i} = 1 \quad (7.25)$$

$$\frac{\partial e_i}{\partial e_j} = 0 \quad (7.26)$$
$$i \neq j$$

for F_{52},

$$\frac{\partial e_i}{\partial \delta_i} = 0 \quad (7.27)$$

$$\frac{\partial e_i}{\partial \delta_j} = 0 \quad (7.28)$$
$$i \neq j$$

In summary, the above equations express each of the measurements in terms of the state variables and the partial derivatives of each of the measurements with respect to each of the state variables. These equations can now be used to find the state variables from the measurements via a Newton-Raphson iterative technique.

VIII. MEASUREMENTS REQUIRED

The purpose here is not necessarily to determine the optimum metering configuration, since this is a problem at least equal in magnitude to the sensitivity problem itself,[1] but to consider some of the factors which govern the choice of measurements. As noted earlier, the advantages of weighted least-squares estimation depend upon redundancy being present in the metering scheme. Before considering redundancy however, it is logical to determine the minimum number of measurements that will be required and build from there. It should be borne in mind that in an actual application, there will already be some existing metering configuration, and it will be necessary to start from this in designing a state estimator.

The following types of measurements will be considered:

1. Real and reactive power injected at a bus
2. Real and reactive line flow power
3. Magnitudes of bus voltages

For n buses, there are 2n-1 state variables to be determined. Therefore, to obtain a unique answer for the linearized model, at least 2n-1 measurements will be required if every state variable is to be determined. There are some additional requirements which must be met however, and certain economic factors should be observed.

There are four basic considerations that will govern our choice of measurements, and these can be listed as follows:

---

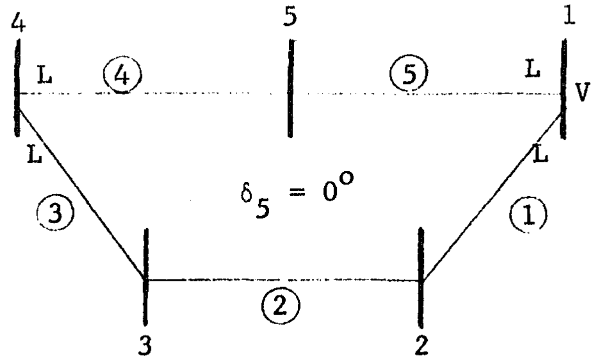
[1] See Reference 4, for example.

1. Measurements are not required at every bus in the system. This characteristic is sometimes referred to as "probing" in the literature; see reference 7 for example.
2. To determine a state variable, it is necessary that it appear in the equation for an available measurement.
3. It is possible to find a solution for the state variables of part of the system without solving for the entire system.
4. To determine the phase relationship between two different sections of the system, the sets of measurement equations for the two sections must be "coupled" in some fashion, i.e., both sets must have at least one state variable in common.

Each of the above characteristics can now be discussed along with some illustrative examples.

To decrease communications and metering costs it is desirable to limit the number of buses at which measurements are taken. For example, as indicated in Figure 8.1(b), all of the state variables can be found from line flow and voltage measurements located at buses 1 and 4. From the same equations it can also be seen that all of the state variables are included and that the equations cannot be divided into any two groups that do not have at least one state variable in common, i.e., coupling will always exist between any two complete sets.

Figure 8.1(c) indicates that measurements at buses 1 and 2 are adequate for determining all of the state variables except e_4 and δ_4.



1 = Bus Number

(1) = Line Number

V = Voltage Measurement

L = Line Flow Measurement

(a) Network

$$
\left[\begin{array}{ccccc|cccc}
a & - & - & - & a & a & - & - & - \\
a & a & - & - & - & a & a & - & - \\
- & - & - & a & a & - & - & - & a \\
- & - & a & a & - & - & - & a & a \\
\hline
 & & \text{same as} & & & \text{above} & & & \\
 & & & & & & & & \\
 & & & & & & & & \\
 & & & & & & & & \\
\hline
a & - & - & - & - & - & - & - & -
\end{array}\right]
\begin{bmatrix}
\Delta e_1 \\ \Delta e_2 \\ \Delta e_3 \\ \Delta e_4 \\ \Delta e_5 \\ --- \\ \Delta\delta_1 \\ \Delta\delta_2 \\ \Delta\delta_3 \\ \Delta\delta_4
\end{bmatrix}
=
\begin{bmatrix}
\Delta g_{15} \\ \Delta g_{12} \\ \Delta g_{45} \\ \Delta g_{43} \\ ---- \\ \Delta h_{15} \\ \Delta h_{12} \\ \Delta h_{45} \\ \Delta h_{43} \\ ---- \\ \Delta e_1
\end{bmatrix}
$$

(b) Equations for measurements at buses 1 and 4 only

The equations shown are intended to represent how the incremental change in each measurement on the right depends upon the incremental change in each state variable on the left. An a is used to represent any non-zero term in the coefficient matrix.

Figure 8.1. A five bus system

$$
\left[\begin{array}{cccc|ccc}
a & - & - & a & a & - & - \\
a & a & - & - & a & a & - \\
- & a & a & - & - & a & a \\
\hline
\multicolumn{4}{c|}{\text{same as}} & \multicolumn{3}{c}{\text{above}} \\
 & & & & & & \\
 & & & & & & \\
\hline
a & - & - & - & - & - & -
\end{array}\right]
\left[\begin{array}{c}
\Delta e_1 \\ \Delta e_2 \\ \Delta e_3 \\ \Delta e_5 \\ \hline \Delta\delta_1 \\ \Delta\delta_2 \\ \Delta\delta_3
\end{array}\right]
=
\left[\begin{array}{c}
\Delta g_{15} \\ \Delta g_{12} \\ \Delta g_{23} \\ \hline \Delta h_{15} \\ \Delta h_{12} \\ \Delta h_{23} \\ \hline \Delta e_1
\end{array}\right]
$$

(c) Equations for measurements at buses 1 and 2 only

$$
\left[\begin{array}{ccc|cc}
a & - & a & a & - \\
a & a & - & a & a \\
\hline
a & - & a & a & - \\
a & a & - & a & a \\
\hline
a & - & - & - & -
\end{array}\right]
\left[\begin{array}{c}
\Delta e_1 \\ \Delta e_2 \\ \Delta e_5 \\ \hline \Delta\delta_1 \\ \Delta\delta_2
\end{array}\right]
=
\left[\begin{array}{c}
\Delta g_{15} \\ \Delta g_{12} \\ \hline \Delta h_{15} \\ \Delta h_{12} \\ \hline \Delta e_1
\end{array}\right]
, \quad \delta_5 = 0^{o},
$$

$$
\left[\begin{array}{ccc}
a & a & a \\
a & a & a \\
a & - & - \\
- & a & -
\end{array}\right]
\left[\begin{array}{c}
\Delta e_3 \\ \Delta e_4 \\ \Delta\delta_4 \\
\end{array}\right]
=
\left[\begin{array}{c}
\Delta g_{43} \\ \Delta h_{43} \\ \Delta e_3 \\ \Delta e_4
\end{array}\right]
, \quad \delta_3 = 0^{o}
$$

(d) Equations for measurements at buses 1 and 4 that can be divided into two uncoupled groups.

Figure 8.1. Continued

Figure 8.1(d) indicates two sets of measurements taken at buses 1 and 4. In this case there are nine measurements and nine variables, but no coupling exists between the two sets of equations so the phase relationship between each set cannot be determined. In this case each set must have its own reference angle (δ_3 and δ_5 for example), and then each can be solved independently of the other. Note that the second set contains more equations than independent variables, so a solution does not necessarily exist unless an equation is omitted or a weighted least squares approach is used. This type of uncoupled solution is mentioned only as a possibility, since it may or may not provide very useful information in an actual physical application.

It is interesting to make some further general observations from this example. If line flow measurements are used, two different measurements can be obtained for each line connected to a particular bus. Bus injection measurements, however, provide a total of only two different measurements, regardless of how many lines are connected to the bus. Also note that a voltage magnitude measurement equation contains only one variable and therefore provides no coupling between two sets of equations (the measurement can always be grouped with the set that contains that particular voltage). Therefore since it is desirable to limit the number of buses at which measurements are taken, line flow measurements appear to have an economic advantage since they should have a higher information content to cost ratio.

It should be noted that although items 2 and 4 in the previous list are necessary conditions for obtaining a solution, they are not sufficient. We still have no guarantee that $(F'R^{-1}F)$ will be well conditioned or even nonsingular, as was previously assumed. These problems did not arise in the experimental system discussed in Section XII, but they are distinct possibilities and should be kept in mind.

IX. SOURCES OF ERROR

The model to be used for the weighted least-squares estimator contains a wide variety of parameters and each of these should be examined as a possible error source. In examining the effects of a particular parameter, it is also necessary to determine the magnitude of the error that is likely to be involved. Naturally, some parameters are more likely to be in error than others since the accuracy of the available data will probably not be uniform (for example line impedances will undoubtedly be more accurate than the variance of the measurement errors). The relative effect of the parameters can also be expected to vary, i.e., a parameter with a close tolerance may still cause larger errors than one that is known only approximately. Another of the primary reasons for conducting a study of this type is to determine where simplifications can be made in the computer program and the measurement system (is it necessary to monitor tap positions of transformers, account for transmission lines that are open at one end only as in Appendix B, etc.?). In an initial study of this nature there are likely to be some sources of error which will be overlooked, and one cannot expect to investigate every possible combination of errors that could arise, but keeping this in mind, we can proceed with what information is available.

A. Transmission Line Impedances

The first parameters to be considered are the transmission line impedances. If the power levels in each of the three phases are assumed to be balanced, a single phase equivalent circuit can be used, and the equivalent impedance is the same as the positive sequence impedance of symmetrical component analysis.[1] If the transmission line is less than 150 miles in length[2] the equations for the lumped series resistance and inductance and the shunt capacitance and conductance can be written as follows:

Series resistance:[2]

$$R = r \cdot \ell \text{ ohms} \tag{9.1}$$

where r = resistance per unit length of the wire material (ohms/mile)

ℓ = length (miles)

Series inductance of a single phase of a three phase, completely transposed line:[2]

$$L = 0.3219 \ln \frac{D_m}{D_s} \cdot \ell \text{ mh.} \tag{9.2}$$

where $D_m = (D_{ab}\, D_{bc}\, D_{ca})^{1/3}$

D_{ab} = center-to-center distance between phase a and b

$D_S = (D_{S1}\, D_{S2}\, D_{S3})^{1/3}$

D_{S1} = self GMD (geometric mean distance) of phase a in position 1 of a transposition

= (0.7788) $\cdot$ radius for cylindrical wire

ℓ = length (miles)

---

[1](28, p. 158).

[2](1, p. 4.3).

Shunt capacitance to neutral of a single phase of a three phase, completely transposed line:[1]

$$C_n = \frac{0.0894}{\ln(D_m/r)} \cdot \ell \quad \text{ufd.} \tag{9.3}$$

where D_m = same as above

r = wire radius

ℓ = length (miles)

Shunt conductance: negligible.[2]

The equations for the parameters of other types of transmission lines (such as those containing bundled conductors, parallel circuits, etc.) will vary somewhat from the above equations, but for balanced conditions, each is similar in form to the equations shown here. Note that the accuracy of these equations is primarily a function of the line length, ℓ, since all constants are well known physical quantities that have a slight variation over the normal temperature range and all other dimensions appear in the argument of a logarithm. Therefore it the length of the line is accurately known it should be possible to determine accurate line parameters.

No experimental data on the accuracy of Equations 9.1, 9.2, and 9.3 was directly available, but reference 15 includes some experimental results based on a study of a 154 Kv transmission line of 270 miles in length. These results are repeated in Table 9.1. Due to the length of

[1](1, p. 5.2).

[2](28, p. 96).

this line, exact long line formulas[1] were used for these calculated values, but the errors between the calculated and experimental results may give some indication of the errors that would result in using Equations 9.1, 9.2, and 9.3 for shorter lines.

Table 9.1. Calculation errors in transmission line parameters (from reference 15)

	Test Data	Source 1[a] Calculations	Source 2[a] Calculations	Max. % Error
Resistance (ohms)	68.7	63.0	64.3	8.3%
Inductance (mh)	0.590	0.575	0.576	2.5%
Capacitance (ufd)	3.68	3.74	3.74	1.6%

[a]Note: Source 1 and 2 refer to different sources of calculated data in reference 15.

Based on such a limited amount of data, we cannot be certain that these results will necessarily be typical for every case, but this information along with our knowledge about the accuracy of the terms in Equations 9.1, 9.2, and 9.3 seems to indicate that the parameters can be determined fairly accurately.

[1](28, pp. 109-111).

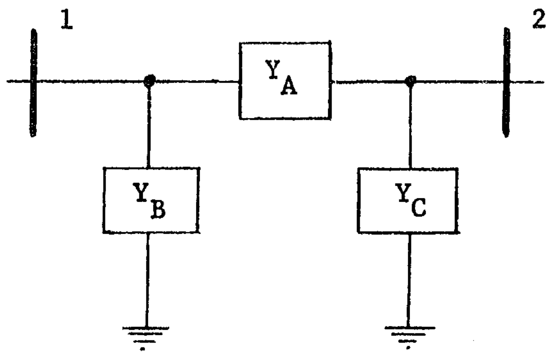
B. Off-Nominal Transformer Tap Settings

Each circuit in an electrical power transmission network will have some nominal voltage associated with it, such as 345 kV., 161 kV., 69 kV., etc. In the per unit system of network calculations it is customary to use this nominal voltage as the base voltage for a particular circuit. Thus, if all transformers have winding ratios equal to the ratio of the nominal voltages on the primary and secondary sides, the transformer can be represented by its per unit series impedance and an ideal transformer with a turns ratio = 1. In an actual physical system however, these transformer ratios will frequently differ from the nominal value. There are various reasons for this, one being that transmission lines may be connected in a loop, and losses in the system will cause voltage differences which will result in circulating currents unless these voltages are compensated for by the transformer ratios. Another possibility is the case where it is desirable to regulate the voltage by means of a tap changing under load (TCUL) transformer, where a stepping switch automatically changes the tap position to maintain a constant voltage on one winding. The position of this tap may or may not be available for use in the on-line state estimation program, so it should be investigated as a possible source of error.

In the per unit system, the nominal value of a transformer ratio = 1, and the typical range for a TCUL transformer is 0.85 to 1.15 in steps of 0.00625.[1] For the state estimation program, all

[1](1, p. 7.52).

transformers may be represented by the model shown in Figure 9.1.



Y_T = transformer per unit admittance

a = per unit winding ratio = $\frac{V_2}{V_1}$ (@ no load)

$Y_A = aY_T$

$Y_B = a(a-1)Y_T$

$Y_C = (1-a)Y_T$

Figure 9.1. Transformer representation

In the event that no loads are connected to buses 1 and 2 in Figure 9.1, this model will add two extra buses and four extra states to the system. We can easily compensate for this however since the following bus power injection equations can be used as measurements:

$$p_1 + jq_1 = 0, \quad p_2 + jq_2 = 0 \tag{9.4}$$

Thus we have added four states and four measurements to the system. This of course increases the dimension of the problem, but it does simplify the computer program, especially in the case where "a" may be changing with time.

Using the model in Figure 9.1, it will then be possible to study how errors in "a" will affect the state estimates and to determine if this quantity should be monitored for TCUL transformers.

C. Errors in the Measurement Error Covariance Matrix

Of all the parameters considered in this study, this undoubtedly will be the most inaccurate. The task of collecting adequate data for determining accurate covariance terms will probably be an enormous one, and it is very likely that much of this information will be only a rough approximation. For example, if the variance of a measurement is assumed to be 3% when it is actually 2% we have a +50% error in the value of this term. Measurement errors here refer to the total error between the actual quantity being measured and the reading that is fed into the computer. Among the sources that can contribute to this are,

1. Meter errors, which are typically characterized by a bias
2. Transmission errors which vary with time
3. Analog to digital conversion errors which vary with time

As mentioned in Section II, references 5 indicates that bias errors tend to dominate the total error. However for the weighted least-squares estimator, the time behavior of the errors is not important since the dynamics of the system are not taken into account.

There are various ways that data on the covariance terms could be obtained, and it is appropriate to discuss a few of these at this point. If any type of maintenance program is in effect, it seems that this might be a good source of data since the error in each instrument could be checked before it is recalibrated. Over a period of time this could provide a considerable amount of field operating data. Manufacturer's data on the particular instruments involved should also be an important source of information. To achieve accurate results it may also be necessary to perform some special tests on the existing instrumentation, but it is probably desirable to hold these to a minimum because of the extensive effort that may be involved.

The data available to the author on these errors is rather limited, but reference 5 does provide some data obtained by Systems Control, Incorporated, and this is repeated below:

$$\sigma_V \simeq 0.0025 \text{ to } 0.003$$

$$\sigma_{MW} \simeq 0.006\ MW_{input} + 0.002\ MW_{full\ scale}$$

$$\sigma_{MVAR} \simeq \sigma_{MW}$$

where σ_V = standard deviation of the voltage measurement errors

σ_{MW} = standard deviation of real power measurement errors

σ_{MVAR} = standard deviation of reactive power measurement errors

$MW_{full\text{-}scale}$ is 5.0 for 230 kV. lines, 2.5 for 115 kV. lines, and MW_{input} is the actual flow quantity (in p.u.).

This data includes no information on the accuracy of the above terms, so we will be forced to depend quite heavily on our own judgement in determining this. These equations do provide an indication of the magnitude of the terms involved however, and the information contained therein is certainly better than none at all.

D. Large Measurement Errors

The problem of measurement errors far in excess of those normally expected has already been investigated in reference 14 in conjunction with a special algorithm to suppress the effects of these errors. There is little doubt that this problem must be accounted for in an on-line state estimation system, but it may be just as efficient to compensate for this problem by imposing limits on the allowable range of the input data. Such an arrangement could be implemented by imposing limits either when the data is received at the control center or when it is fed into the computer.

Even if a limiting scheme is employed, it must be determined how such data will affect the state estimates since these measurements will still contain errors considerably larger than expected. A similar problem exists for those measurements which contain errors which are larger than expected but not large enough to reach the bounds of a limiter. A typical example might be a voltage measurement with a +10% error where an error of less than $\pm 2\%$ was expected.

X. DESCRIPTION OF EXPERIMENT

Having formulated a procedure for evaluating the effects of modeling errors in Section VI, we now desire to apply these methods to an actual power network. Hopefully an experiment of this nature should 1) indicate the sensitivity of the estimator to the various parameters of the model, and 2) uncover some of the problems that arise in actual physical applications.

The experiment was performed by using a digital computer to simulate an on-line state estimation program and then analyzing the estimates that result when various modeling errors are present. In the process of conducting this analysis, it was necessary to obtain the following information:

1. From simulated measurements, determine the optimum estimate and its variance.
2. Introduce modeling errors and determine the resulting state estimate, its variance, and the expected value of the error.
3. Compare the results of steps 1 and 2 to determine the effects of such modeling errors upon the estimates.
4. Using the results of steps 1 and 2, calculate various unmeasured power levels, to determine the sensitivity of such calculations to errors in the state estimates.

Several steps were required to obtain the necessary data, and these are presented in the following outline form before going into greater detail:

1. The set of variables to be measured, $\underline{f}(\underline{x})$ (refer to Equation 3.1), were selected.

2. All network parameters needed to find $\underline{f}(\underline{x}_o)$ and $F(\underline{x}_o)$ (refer to Equation 3.5) were obtained directly from Iowa Power and Light Company (IPALCO).

3. The covariance matrix, R, of the measurements, $\underline{Z}$, was determined.

4. The state, $\underline{x}$, defined to be the true state was obtained from a load flow program using scheduled bus injections supplied by IPALCO.

5. The simulated measurements (refer to Equation 3.1) were found by calculating

$$\underline{Z} = \underline{f}(\underline{x}) + \underline{0}$$

6. Storage location codes were generated to handle the sparse matrices involved in the computer programs.

7. For the correct model, solve for the optimum estimate, $\hat{\underline{X}}$, (refer to Equation 3.18)

$$(F'(\underline{x}_o)R^{-1}F(\underline{x}_o))\hat{\underline{X}} = F'(\underline{x}_o)R^{-1}(\underline{Z} - \underline{f}(\underline{x}_o) + F(\underline{x}_o)\underline{x}_o) \ .$$

8. Find the variance of $\hat{\underline{X}}$ by finding the diagonal terms of its covariance matrix, $P(\underline{x}_o)$, (refer to Equation 4.1)

$$P(\underline{x}_o) = (F'(\underline{x}_o)R^{-1}F(\underline{x}_o))^{-1} \ .$$

9. Introduce modeling errors and calculate the resulting estimate, $\hat{\underline{X}}_c$, (refer to Equation 5.2)

$$(F_c'(\underline{x}_{oc})R_c^{-1}F_c(\underline{x}_{oc}))\hat{\underline{X}}_c = F_c'(\underline{x}_{oc})(\underline{Z} - \underline{f}_c(\underline{x}_{oc}) + F_c(\underline{x}_{oc})\underline{x}_{oc})$$

10. Find the expected error of $\hat{\underline{X}}_c$, $E[(\hat{\underline{X}}_c - \underline{x})/\underline{x}]$, (refer to Equation 5.4)

$$E[(\hat{\underline{X}}_c - \underline{x})/\underline{x}] = (F_c'(\underline{x}_{oc})R_c^{-1}F_c(\underline{x}_{oc}))^{-1}F_c'(\underline{x}_{oc})R_c^{-1}[\underline{f}(\underline{x}_o) - \underline{f}_c(\underline{x}_{oc}) + F(\underline{x}_o)(\underline{x} - \underline{x}_o)] - (\underline{x} - \underline{x}_{oc})$$

11. Find the variance of $\hat{\underline{X}}_c$ by finding the diagonal terms of the actual covariance matrix of $\hat{\underline{X}}_c$, $P_a(\underline{x}_{oc})$, (refer to Equation 4.4)

$$P_a(\underline{x}_{oc}) = (F_c'(\underline{x}_{oc})R_c^{-1}F(\underline{x}_{oc}))^{-1}F_c'(\underline{x}_{oc})R_c^{-1}RR_c^{-1}F_c(\underline{x}_{oc}) \cdot (F_c'(\underline{x}_{oc})R_c^{-1}F_c(\underline{x}_{oc}))^{-1}$$

12. Evaluate the effects of the modeling errors on the estimates by using the results of steps 8, 10, and 11 as criteria. That is, 1) examine $E[(\hat{\underline{X}}_c - \underline{x})/\underline{x}]$ to determine the magnitude of the average error, and 2) compare the diagonal terms of P_a with those of P to determine how the actual variance differs from that of the optimum estimate.

13. Calculate certain selected unmeasured line flow power levels using $\underline{x}$.

14. Calculate the same line flows as in 13 using

$$\underline{x} + E[(\hat{\underline{X}}_c - \underline{x})/\underline{x}]$$

15. Compare the results of steps 13 and 14 to determine the effects of estimate errors on these calculations.

To maintain computation time and storage requirements at a reasonable level, it was necessary to utilize the sparsity of all matrices involved in the computer programs. However, since this experiment was intended as an off-line study, absolute optimum speed and storage were considered to be of secondary importance so that more effort could be concentrated on the sensitivity analysis. As a result, the computation time and storage requirements of the STATE ESTIMATOR program (approximately 6.5 seconds/iteration and 120 K bytes of memory for the IBM 360/65) could undoubtedly be reduced for on-line applications. These requirements are approaching a reasonable level for on-line use however, and there is reason to believe that use of the following techniques could improve these specifications considerably:

1. Machine language instead of Fortran.
2. Optimum ordering in the Gaussian elimination step for solving the simultaneous equations.
3. An improved storage scheme for the sparse matrices.

It should also be stressed that the STATE ESTIMATOR program involved no approximations other than the linearized model itself. One approximation that has been shown to work reasonably well for small systems (reference 23) would be to use the same Jacobian matrix for several iterations. This would not only eliminate the calculation of the Jacobian with each iteration, but it would also save time in the Gaussian elimination process since the upper triangular coefficient matrix would be the same for each iteration and could be stored.

A. Data Preparation

1. Parameter calculation

All transmission line data was supplied by IPALCO in terms of the series impedance and shunt admittance for each line. Nominal tap settings were also supplied for each transformer in the system. Occasionally the data for a transmission line and a transformer in series were combined; in which case, it was assumed that the winding ratio = 1.0 so that the shunt admittance would be the same at each end of the equivalent line (see Section IX-B). All of the appropriate line admittance data was then calculated from this information.

2. Standard deviation of measurement errors

Very little first hand information was available for determining these quantities, so the following formulas from reference 5 were utilized (also see Section IX-C):

$$\sigma_V = 0.0033$$

$$\sigma_{MW} = 0.006 \text{ MW input} + 0.002 \text{ MW full scale}$$

$$\sigma_{MVAR} = \sigma_{MW}$$

where σ_V = Standard deviation of the voltage measurement errors

σ_{MW} = Standard deviation of real line flow power measurement errors

σ_{MVAR} = Standard deviation of reactive line flow power measurement errors

Full scale and approximate input line flow values were obtained from a previous load flow study supplied by IPALCO. The full scale value was set equal to the maximum line rating and the approximate input value was set equal to the value calculated in the load flow study. The same standard deviation was used for both real and reactive components, and the larger of the two readings was always chosen for the calculation. Bus injection measurements were handled in a similar fashion except that the maximum rating was arbitrarily assumed to be twice the average if more than one line connected to the bus.

3. True state of the system

In order to determine the expected error in the estimates it is certainly necessary to know the true state of the system a priori. To establish a value for this true state that would be reasonable from a physical standpoint, the STATE ESTIMATOR program was first run as a load flow program. This was accomplished by using bus injection schedules supplied by IPALCO and using an identity matrix for the measurement inverse covariance matrix. The solution obtained was then defined to be the true state of the system and was recorded for future reference.

4. Simulated measurements

It should be noted that some care must be taken in selecting values for these quantities or complete chaos may result. In a weighted least-squares problem one does not usually expect a solution that fits the data exactly, but the values of the measurements should be at least reasonably consistent with each other. If this is not the case, the estimation program may not converge or may produce answers that are completely ridiculous in a physical sense.

In this experiment, the values of all simulated measurements were calculated directly from the true state of the system, which was known a priori. The primary goal of this study was to determine the relative effects of parameter errors, so no measurement errors were included in this simulation. As pointed out in Section IV, this still represents a valid set of measurements however, since zero is a perfectly valid random error.

B. Storage Location Codes

As mentioned earlier, it was necessary to exploit the sparsity of all matrices involved in order to obtain reasonable computation time and storage requirements. As a result, it was also necessary to generate various integer arrays to instruct the computer where to locate the correct elements during certain operations. These integer arrays were generated in the STORAGE LOCATION program, and the results were then tabulated as data for the STATE ESTIMATOR and SENSITIVITY ANALYSIS programs. This scheme decreased the storage requirements and number

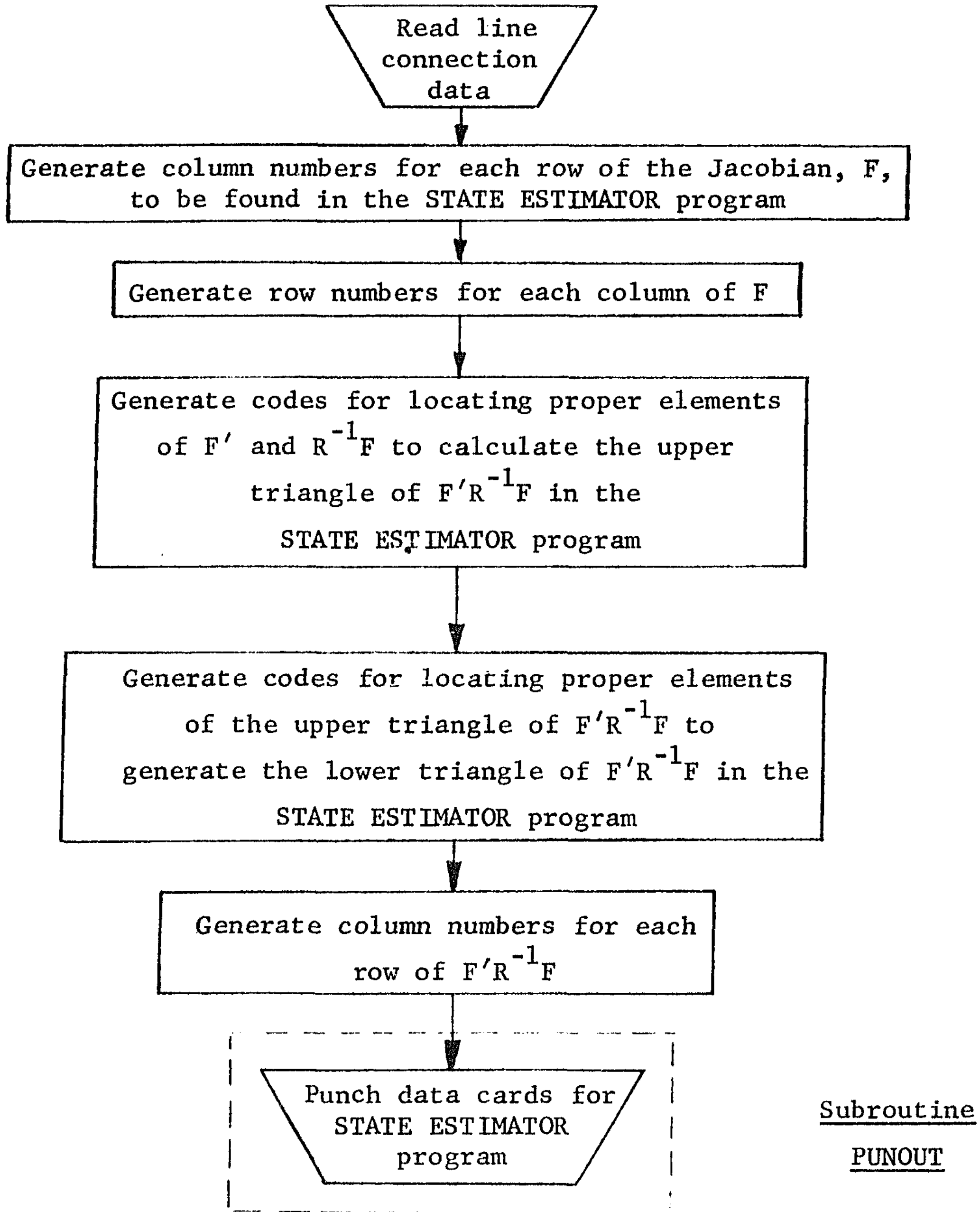


Figure 10.1. STORAGE LOCATION program

of arithmetic operations required, and it also eliminated the necessity of performing any scanning operations to find certain elements in storage. The coding for this program is included in Appendix C, and the operation can be explained as follows by referring to Figure 10.1:

1. Read in all line connection data.
2. Generate the column numbers of each row of nonzero elements of the Jacobian matrix F. This will be used for identifying the nonzero elements of F, which will be calculated in the STATE ESTIMATOR program and stored by rows.
3. Generate the row numbers of each column of nonzero elements of the Jacobian matrix F. This will be used for identifying the nonzero elements of F, which will also be stored by columns in the STATE ESTIMATOR program.
4. Generate codes for locating the proper elements of F' and $R^{-1}F$ to be used for calculating the upper triangle of $F'R^{-1}F$ in STATE ESTIMATOR. To find $F'R^{-1}F$, the computer must know how to find the product of each row of F' and each column of $R^{-1}F$. This step produces codes to be used for locating the proper elements and eliminates the need for scanning in STATE ESTIMATOR.
5. Generate codes for locating the proper elements of the upper triangle of $F'R^{-1}F$ so that the lower triangle may be generated in STATE ESTIMATOR.

6. Generate the column numbers of each row of nonzero elements of $F'R^{-1}F$ (including both upper and lower triangular parts). This information will be necessary for performing the Gaussian elimination and back substitution steps in STATE ESTIMATOR.
7. Punch data cards to be used in the STATE ESTIMATOR and SENSITIVITY ANALYSIS programs.

C. STATE ESTIMATOR Program

This section describes the program of the experiment that calculates the state estimates from the simulated measurements. The coding for this program is included in Appendix D and a flow diagram is shown in Figure 10.2.

The operation of the program can be explained as follows by referring to Figure 10.2:

1. All initial data is first read in and stored.
2. Using this initial data, subroutine CMEAS calculates $\underline{f}(\underline{x}_o)$.
3. The new cost function, $J(\underline{x}_o)$, is set equal to zero.
4. The first set of measurements, $\underline{Z}$, are read in.
5. The old cost function is set equal to the new cost function.
6. The new cost function is re-calculated using the last measurement and the calculated measurements from CMEAS.
7. A check is made to determine if $|\text{New Cost} - \text{Old Cost}|$ is less than some predetermined tolerance. If so, the

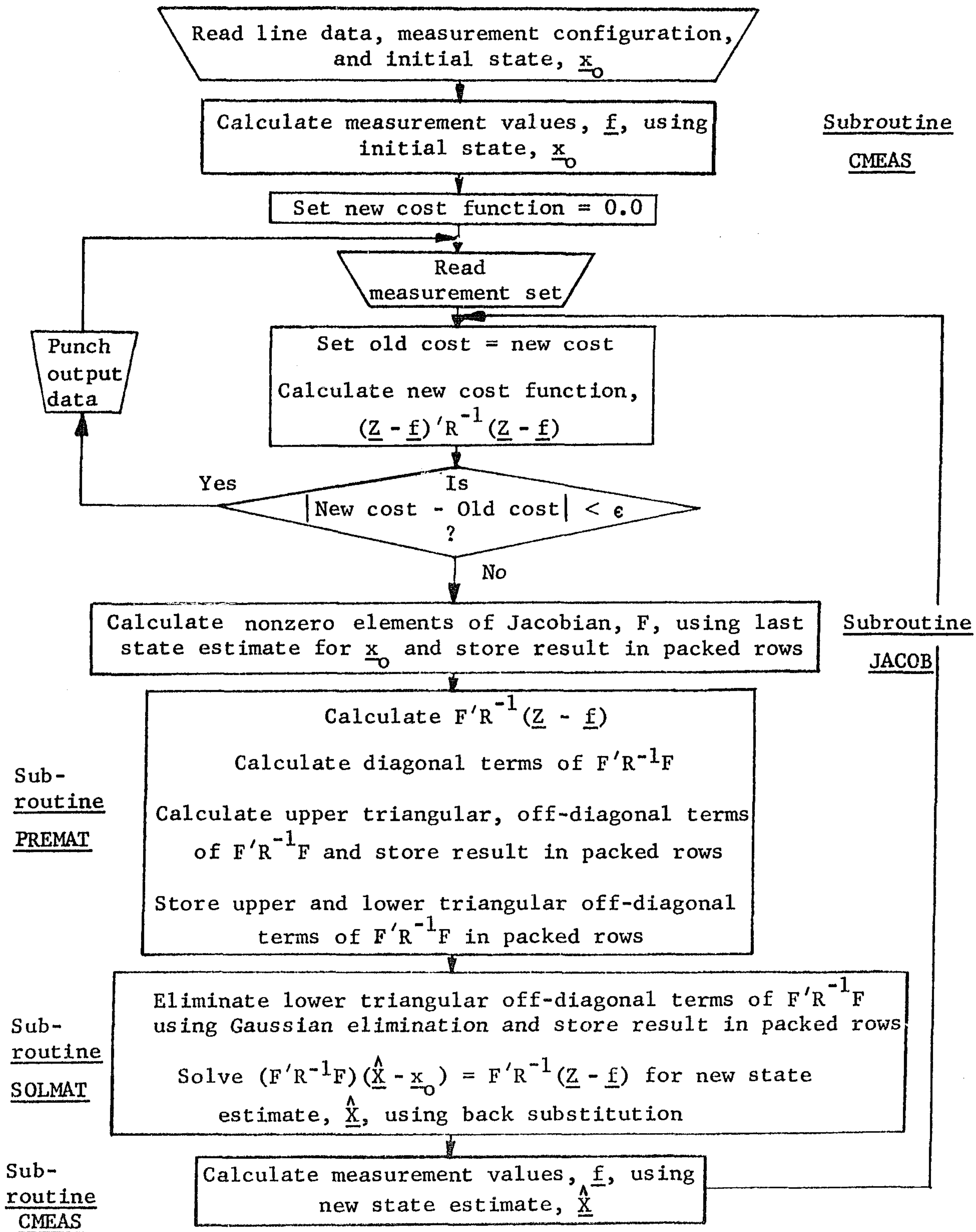


Figure 10.2. STATE ESTIMATOR program

present state estimate is said to be satisfactory; if not, the iteration process is initiated.

8. Assuming that the initial state did not satisfy the convergence tolerance, subroutine JACOB is called, which calculates the Jacobian matrix using the present state estimate for $\underline{x}_o$. Only the predictable nonzero elements are calculated and stored, and the result is referred to as a "packed" row of the Jacobian matrix.

9. Subroutine PREMAT is then called to calculate the matrices of the equation,

$$(F'(\underline{x}_o)R^{-1}F(\underline{x}_o)(\hat{\underline{X}}(k) - \underline{x}_o) = F'(\underline{x}_o)R^{-1}(\underline{Z}(k) - \underline{f}(\underline{x}_o))$$

These matrices are calculated in the order indicated in Figure 10.2.

10. Subroutine SOLMAT is called to solve the equation shown in step 9 by Gaussian elimination and back substitution.

11. Subroutine CMEAS is called to calculate $\underline{f}(\hat{\underline{X}})$ from the new state estimate.

12. Steps 5 through 7 are repeated.

13. If the convergence tolerance of step 7 is satisfied the output data is punched and a new measurement data set is read. If not, steps 8 through 12 are repeated using the last state estimate for $\underline{x}_o$, until a satisfactory state estimate is obtained.

Note that $\hat{\underline{X}}$ is found by solving the set of simultaneous equations indicated by the equation in step 9; no attempt is made to find the inverse of $F'R^{-1}F$ since this calculation involves much more computation time. It is necessary to eventually calculate $(F'R^{-1}F)^{-1}$ since this matrix is needed to determine both the expected error and the variance of the state estimate, but since we are interested only in the $(F'R^{-1}F)^{-1}$ of the last iteration, it is more economical to calculate this in the SENSITIVITY ANALYSIS program.

D. SENSITIVITY ANALYSIS Program

As discussed in Section VI, modeling errors will be evaluated by determining the expected error, optimum variance, and actual variance of the estimates. These terms can be found from Equations 5.4, 4.1, and 4.4 and are repeated below for convenience,

$$E[(\hat{\underline{X}}_c - \underline{x})/\underline{x}] = (F_c'R_c^{-1}F_c)^{-1}F_c'R_c^{-1}[\underline{f} - \underline{f}_c + F(\underline{x} - \underline{x}_o)] - (\underline{x} - \underline{x}_{oc}) \tag{5.4}$$

$$P = (F'R^{-1}F) \tag{4.1}$$

$$P_a = (F_c'R_c^{-1}F_c)^{-1}F_c'R_c^{-1}RR_c^{-1}F_c(F_c'R_c^{-1}F_c)^{-1} \tag{4.4}$$

The program for performing these calculations is included in Appendix E and the flow chart is shown in Figure 10.3. The operation of the program can be explained as follows by referring to Figure 10.3:

1. Read all input data including the true state of the system, $\underline{x}$, all measurement variance data, and the outputs of the

Read input data, including the output from the STATE ESTIMATOR and STORAGE LOCATION programs for the correct and incorrect cases

Subroutine CALELM

Calculate diagonal terms of $F_c' R_c^{-1} F_c$

Calculate upper triangular, off-diagonal terms of $F_c' R_c^{-1} F_c$ and store in packed rows

Store upper and lower triangular off-diagonal terms of $F_c' R_c^{-1} F_c$ in packed rows

---

Calculate $\underline{f} - \underline{f}_c + F(\underline{x} - \underline{x}_o)$

Calculate $F_c' R_c^{-1}$ and $F_c' R_c^{-1} R^{\frac{1}{2}}$

Subroutine REDMAT

Set RHS(1) = 1.0 and eliminate lower triangular, off-diagonal terms of $F_c' R_c^{-1} F_c$ using Gaussian elimination. Store the result in packed rows and record the operations in the arrays PRO, KOL, LAOS, and LAPE

Use back substitution to find the first column of $(F_c' R_c^{-1} F_c)^{-1}$

Calculate the first term of $(F_c' R_c^{-1} F_c)^{-1} F_c' R_c^{-1} [\underline{f} - \underline{f}_c + F(\underline{x} - \underline{x}_o)] - (\underline{x} - \underline{x}_{oc})$

Calculate the first diagonal term of $(F_c' R_c^{-1} F_c)^{-1} F_c' R_c^{-1} R R_c^{-1} F_c (F_c' R_c^{-1} F_c)^{-1}$

For all remaining states, calculate each column of $(F_c' R_c^{-1} F_c)^{-1}$, each term of $(F_c' R_c^{-1} F_c)^{-1} F_c' R_c^{-1} [\underline{f} - \underline{f}_c + F(\underline{x} - \underline{x}_o)] - (\underline{x} - \underline{x}_{oc})$, and each diagonal term of $(F_c' R_c^{-1} F_c)^{-1} F_c' R_c^{-1} R R_c^{-1} F_c (F_c' R_c^{-1} F_c)^{-1}$ Perform this operation using the arrays PRO KOL, LAPS, and LAPE recorded in subroutine REDMAT.

Figure 10.3. SENSITIVITY ANALYSIS program

STORAGE LOCATION and STATE ESTIMATOR programs. This includes data for the correct and incorrect models.

2. Call subroutine CALELM to,
   a) Calculate the diagonal terms of the erroneous covariance matrix $F_c' R_c^{-1} F_c$.
   b) Calculate the upper triangular, off-diagonal, nonzero terms of $F_c' R_c^{-1} F_c$ and store by rows (column locations are supplied by the STORAGE LOCATION program).
   c) Since $F_c' R_c^{-1} F_c$ is symmetric, the lower triangular section can be found from the upper triangular section. All off-diagonal terms are then stored in packed rows.
3. Calculate $[\underline{f} - \underline{f}_c + F(\underline{x} - \underline{x}_o)]$.
4. Calculate $F'_c R_c^{-1}$ and $F_c' R_c^{-1} R^{\frac{1}{2}}$.
5. Call subroutine REDMAT. This subroutine calculates the first column of $F_c' R_c^{-1} F_c$ and records all of the operations necessary for finding the succeeding columns. This is performed by recording each operation of the Gaussian elimination process as follows:
   a) All of the lower-triangular terms of each row are eliminated before proceeding to the next row. The array PRO contains each of these terms and the array KOL records which column it was located in.

b) Arrays LAPS and LAPE record where the terms for each row start and end in the arrays PRO and KOL.

c) These arrays can then be used by the main program to calculate the remaining columns of $F_c'R_c^{-1}F_c$ without recalculating the terms of PRO and KOL.

6. Before returning to the main program, REDMAT goes on to find the first term of

$$(F_c'R_c^{-1}F_c)^{-1}F_c'R_c^{-1}[\underline{f} - \underline{f}_c + F(\underline{x} - \underline{x}_o)] - (\underline{x} - \underline{x}_{oc})$$

and the first diagonal term of

$$(F_c'R_c^{-1}F_c)^{-1}F_c'R_c^{-1}RR_c^{-1}F_c(F_c'R_c^{-1}F_c)^{-1},$$

as indicated in Figure 4.

7. Using the results from REDMAT, each column of $(F_c'R_c^{-1}F_c)^{-1}$, each term of $(F_c'R_c^{-1}F_c)^{-1}F_c'R_c^{-1}[\underline{f} - \underline{f}_c + F(\underline{x} - \underline{x}_o)] - (\underline{x} - \underline{x}_{oc})$, and each diagonal term of $(F_c'R_c^{-1}F_c)^{-1}F_c'R_c^{-1}RR_c^{-1}F_c(F_c'R_c^{-1}F_c)^{-1}$ is calculated. This result provides the following information:

a) The diagonal terms of $(F_c'R_c^{-1}F_c)^{-1}$ are the calculated (and erroneous) values of the variance of each state estimate.

b) Each term of

$$(F_c'R_c^{-1}F_c)^{-1}F_c'R_c^{-1}[\underline{f} - \underline{f}_c + F(\underline{x} - \underline{x}_o)] - (\underline{x} - \underline{x}_{oc})$$

corresponds to the expected value of the error for each state estimate.

c) The diagonal terms of

$$(F_c'R_c^{-1}F_c)^{-1}F_c'R_c^{-1}RR_c^{-1}F_c(F_c'R_c^{-1}F_c)^{-1}$$

are the actual variances of each of the state estimates.

E. Effect of Estimate Errors on Power Calculations

In addition to determining the effects of modeling errors on the state estimates, it is of interest to determine how the estimate errors will affect subsequent power calculations. Comparing the calculated and measured values of measurements used in obtaining the state estimate may produce rather optimistic results since the state estimates are purposely chosen so that these quantities will agree with each other. In other words, the state estimates will tend to be given errors to compensate for the modeling errors in producing a good fit between measured and calculated data. A more realistic test is to compare the actual and calculated values of quantities not used in obtaining the state estimate. Such a test is not only more objective, but it is extremely important since such calculations are perhaps one of the most useful results to be obtained from on-line state estimation. To obtain some typical results, certain unmeasured line flow powers were calculated using the true state, $\underline{x}$, and the true state plus expected error, $\underline{x} + E[(\hat{\underline{X}}_c - \underline{x})/\underline{x}]$. Extensive calculations were judged unnecessary here since only an indication of the relative effects was desired, and no attempt was made to find all of the unmeasured power levels.

XI. EXPERIMENTAL POWER SYSTEM

A single line diagram of the network chosen for the experiment is shown in Figure 11.1 along with the bus names and voltage levels in Table 11.1. This system is a replica of IPALCO's Central Division which is located in and around the vicinity of Des Moines, Iowa. This particular model represents the normal operating configuration of the system and includes 58 buses and 69 lines. All network parameters were calculated directly from data obtained from IPALCO.

A. Measurement Configuration

Since this experiment was intended to simulate an actual physical application as closely as possible, maximum use was made of existing measurements. The choice of additional measurements can be obtained by a variety of methods, provided the requirements of Section VIII are observed. In this particular case line flows and bus injections were added to the existing measurements until all buses were coupled by the measurement equations. This procedure resulted in 16 extra measurements since some of those already in existence were not necessary for coupling. This measurement configuration provided a redundancy of approximately 14%.

The location of the resulting measurements are indicated in Figure 11.1 along with the appropriate measurement code. It is assumed that all bus injection and line flow measurements include both real and reactive components. The following tabulation should give some indication of the instrumentation that would be required for this proposed measurement configuration:

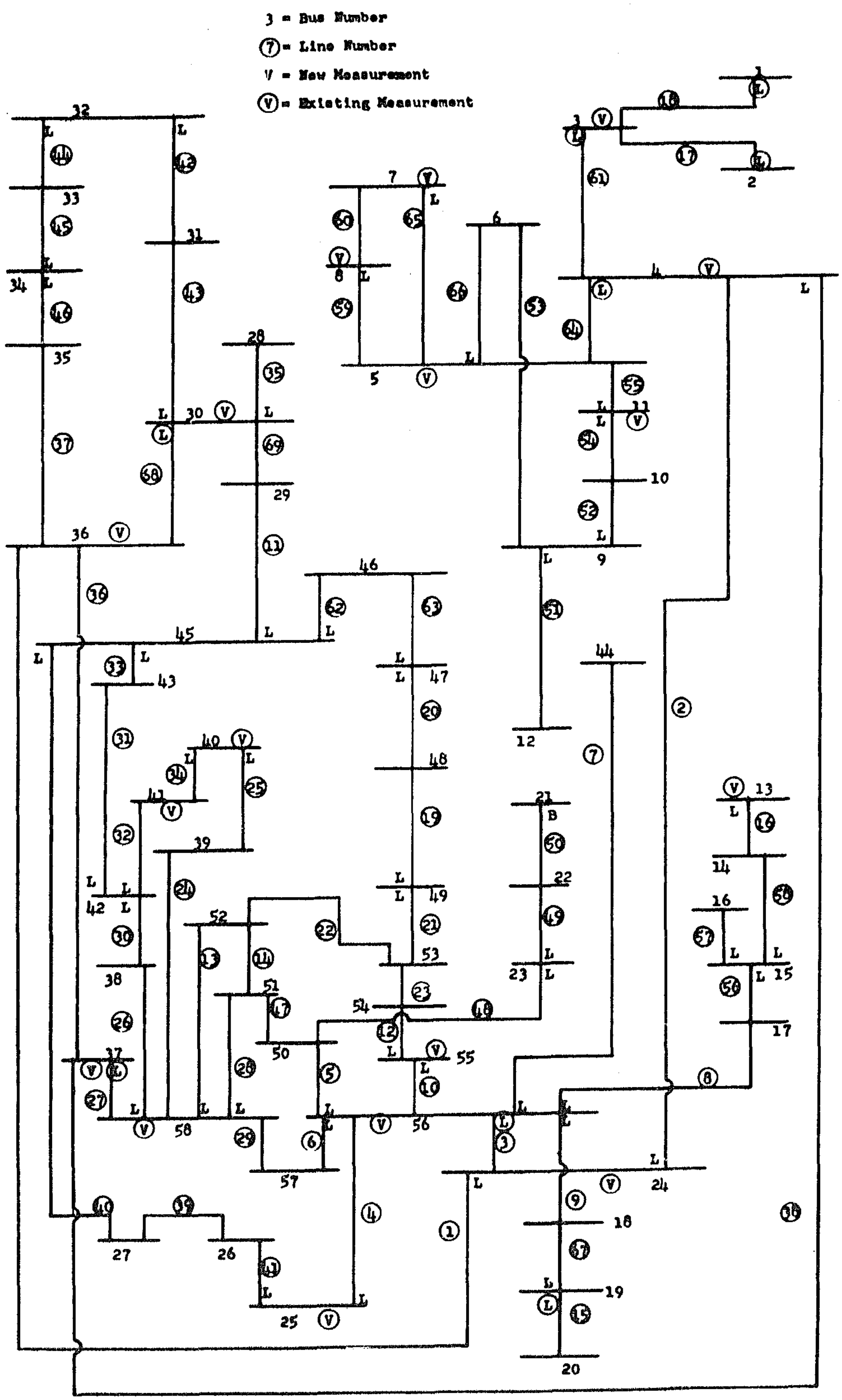


Figure 11.1. IPALCO Central Division

Table 11.1. Bus names and voltage ratings for IPALCO Central Division

Bus No.	Name	Voltage
1	Cooper	345 KV
2	Hills	345
3	Sycamore, 345	345
4	Sycamore, 161	161
5	Sycamore, 69	69
6	John Deere	69
7	30th & Aurora	69
8	76th & Douglas	69
9	E. 22nd & Broadway	69
10	Highland Park	69
11	Firestone	69
12	E. 29th & Hubbell	69
13	Oskaloosa	69
14	Monroe	69
15	Pr. City	69
16	Colfax	69
17	S. E. 124th	69
18	Pleasantville	69
19	Knoxville	69
20	Chariton	69
21	E. 17th & Washington	69
22	23rd & Dean	69

Table 11.1. Continued

Bus No.	Name	Voltage
23	Armstrong, 69	69 KV
24	DPS. 2, 161	161
25	South Des Moines	69
26	Marquette	69
27	63rd & Park	69
28	73rd & Buff	69
29	Penn. - Dixie	69
30	Ashawa, 69	69
31	Shuler	46
32	Adel	46
33	Redfield	46
34	Earlham, 46	46
35	Earlham, 161	161
36	Ashawa, 161	161
37	16th & Wabash, 161	161
38	Waterworks	69
39	16th & Park	69
40	16th & College	69
41	38th & Franklin	69
42	28th & Rock Island	69
43	37th & Rock Island	69
44	46th & Jefferson	69

Table 11.1. Continued

Bus No.	Name	Voltage
45	West Des Moines	69 KV
46	58th & Franklin	46
47	38th & Fagen	46
48	25th & College	46
49	2nd & Clark	46
50	E. 23rd Tap	69
51	River Hills Tap	69
52	River Hills, 69	69
53	River Hills, 46	46
54	Armstrong, 46	46
55	DPS. 2, 46	46
56	DPS. 2, 69	69
57	S. E. 8th Tap	69
58	16th & Wabash, 69	69

- Bus Voltage Measurements -

  Existing = 17

  Necessary Additions = 0

- Bus Injection Measurements -

  Existing = 0

  Necessary Additions = 2

- Line Flow Measurements -

  Existing = 16

  Necessary Additions = 96

- Buses With Instrumentation -

  Existing = 19

  Necessary Additions = 9

Each real and reactive power measurement is counted as a separate measurement, but it should be emphasized that the instrumentation required to obtain one from the other is quite modest. It should also be noted that this configuration is intended only as a reasonable measurement scheme for evaluating the sensitivity analysis and is not necessarily optimum in terms of either cost or accuracy.

B. Electrical Parameters

Table 11.2 is a listing of the series impedances and half of the shunt admittance due to line capacitance for each line in the system. All real shunt admittances proved to be quite small for this system and therefore were omitted from the model. Some of the lines in

Table 11.2 are actually transformers and have been designated by the "TF" if the winding ratio is fixed and by a "TC" if the transformer is a tap changing type. Other lines represent the combination of a transformer with winding ratio = 1.0 in series with a transmission line and are designated by a "TL". All TF and TC transformers are also listed in Table 11.3 along with the nominal winding ratio that was used in determining the true state of the system.

All data is shown in per unit (pu) quantities which are referenced to a base VA of 100 MVA and the base voltage levels in Table 11.1.

Table 11.2. Line parameters for IPALCO Central Division

TF = Transformer with fixed winding ratio

TC = Tap changing transformer (also designated as TCUL)

TL = Transformer and line combined (winding ratio = 1.0)

Head, Tail = Numbers of the buses at each end of the line

All impedances and admittances are given in per unit (pu)

Line	Head	Tail	Series Resistance	Series Reactance	Half Shunt Admittance
1	24	36	0.117E-01	0.493E-01	0.115E-01
2	24	4	0.540E-02	0.440E-01	0.115E-01
3 TF	56	24	0.0	0.350E 00	0.0
4	56	25	0.414E-01	0.104E 00	0.900E-03
5	56	50	0.269E-01	0.683E-01	0.600E-03
6	56	57	0.246E-01	0.625E-01	0.500E-03
7	56	44	0.269E-01	0.682E-01	0.600E-03
8	56	17	0.111E 00	0.185E 00	0.140E-02
9	56	18	0.185E 00	0.324E 00	0.250E-02
10 TF	55	56	0.0	0.154E 00	0.0
11	45	29	0.460E-02	0.190E-01	0.200E-03
12 TL	55	54	0.414E-01	0.211E 00	0.200E-03
13	52	58	0.580E-02	0.236E-01	0.170E-02
14	52	51	0.240E-02	0.690E-02	0.250E-02
15	20	19	0.269E 00	0.454E 00	0.340E-02

Table 11.2. Continued

Line	Head	Tail	Series Resistance	Series Reactance	Half Shunt Admittance
16	13	14	0.558E 00	0.575E 00	0.380E-02
17	2	3	0.610E-02	0.577E-01	0.464E 00
18	1	3	0.780E-02	0.740E-01	0.596E 00
19	48	49	0.306E-01	0.723E-01	0.100E-03
20	48	47	0.192E-01	0.484E-01	0.100E-03
21	49	53	0.103E-01	0.464E-01	0.100E-03
22 TF	53	52	0.0	0.147E 00	0.0
23	53	54	0.460E-01	0.103E 00	0.160E-02
24	39	58	0.490E-02	0.125E-01	0.100E-03
25	39	40	0.530E-02	0.185E-01	0.210E-02
26	58	38	0.220E-02	0.920E-02	0.100E-03
27 TC	58	37	0.0	0.817E-01	0.0
28	58	51	0.121E-01	0.306E-01	0.300E-03
29	58	57	0.279E-01	0.710E-01	0.600E-03
30	38	42	0.160E-02	0.670E-02	0.100E-03
31	42	43	0.320E-02	0.128E-01	0.100E-03
32	42	41	0.122E-01	0.440E-01	0.500E-03
33	43	45	0.102E-01	0.341E-01	0.300E-03
34	41	40	0.760E-02	0.264E-01	0.300E-03
35	28	30	0.256E-01	0.726E-01	0.700E-03
36	36	37	0.230E-02	0.177E-01	0.470E-02
37	36	35	0.173E-01	0.728E-01	0.170E-01
38	37	4	0.350E-02	0.286E-01	0.780E-02

Table 11.2. Continued

Line	Head	Tail	Series Resistance	Series Reactance	Half Shunt Admittance
39	27	26	0.730E-02	0.185E-01	0.200E-03
40	27	45	0.650E-02	0.173E-01	0.100E-03
41	26	25	0.217E-01	0.551E-01	0.500E-03
42	31	32	0.195E 00	0.290E 00	0.400E-03
43 TL	31	30	0.141E 00	0.479E 00	0.400E-03
44	32	33	0.127E 00	0.302E 00	0.500E-03
45	33	34	0.119E 00	0.285E 00	0.500E-03
46 TC	34	35	0.0	0.442E 00	0.0
47	51	50	0.119E-01	0.308E-01	0.200E-03
48	50	23	0.220E-02	0.340E-02	0.0
49	23	22	0.450E-02	0.690E-02	0.100E-03
50	22	21	0.139E-01	0.323E-01	0.300E-03
51	12	9	0.235E-01	0.446E-01	0.400E-03
52	9	10	0.106E-01	0.270E-01	0.200E-03
53	9	6	0.670E-01	0.124E 00	0.100E-02
54	10	11	0.290E-02	0.740E-02	0.100E-03
55	11	5	0.150E-01	0.604E-01	0.500E-03
56	17	15	0.756E-01	0.126E 00	0.100E-02
57	15	16	0.533E-01	0.899E-01	0.700E-03
58	15	14	0.111E 00	0.163E 00	0.110E-02
59	8	5	0.175E-01	0.997E-01	0.100E-02
60	8	7	0.147E-01	0.372E-01	0.300E-03

Table 11.2. Continued

Line	Head	Tail	Series Resistance	Series Reactance	Half Shunt Admittance
61 TF	4	3	0.0	0.190E-01	0.0
62 TL	46	45	0.435E-01	0.207E 00	0.200E-03
63	46	47	0.312E-01	0.768E-01	0.100E-03
64 TC	5	4	0.0	0.409E-01	0.0
65	5	7	0.256E-01	0.855E-01	0.700E-03
66	5	6	0.238E-01	0.812E-01	0.100E-02
67	19	18	0.103E 00	0.179E 00	0.140E-02
68 TC	30	36	0.0	0.119E 00	0.0
69	30	29	0.710E-02	0.264E-01	0.200E-03

Table 11.3. Nominal transformer ratios for IPALCO Central Division

$$\text{Ratio} = \frac{V_{head}}{V_{tail}}$$

TF = Fixed Ratio

TC = Tap Changing Transformer

Head, Tail = Numbers of the buses on each side of the transformer

Transformer	Head	Tail	Ratio
3 TF	56	24	1.025
10 TF	55	56	0.975
22 TF	53	52	0.975
27 TC	58	37	1.010
46 TC	31	30	0.978
61 TF	4	3	1.000
64 TC	5	4	0.997
68 TC	30	36	1.009

XII. EXPERIMENTAL RESULTS

For identification purposes in the tables of data and computer programs the following designations have been used for the state variables:

States 1 through 58 = Voltage magnitudes at buses 1 through 58.

States 59 through 115 = Phase angles at buses 1 through 57 (The phase angle at bus 58 is defined to be zero).

All measurements have been numbered sequentially in the following order:

1. Real bus power injections
2. Reactive bus power injections
3. Real line flow power levels
4. Reactive line flow power levels
5. Voltage magnitudes

All phase angles are expressed in radians, and all other data is expressed in per unit (pu) quantities. The base values used are 100 MVA for the VA base, and the nominal voltage levels shown in Table 11.1 are used for the voltage bases.

A. Standard Deviation of Measurements

Table 12.1 lists the standard deviation for each of the measurements indicated in Figure 11.1. These are assumed to be the correct values for purposes of the sensitivity analysis.

Table 12.1. True standard deviation of each measurement

All values are shown in per unit (pu)

All real and reactive power measurements are assumed to have the same standard deviation

All measurement locations are shown in Figure 11.1

Bus	Std. Dev.
21	0.100E-02

Line Pwr.	Std. Dev.	Line Pwr.	Std. Dev.
1	0.870E-02	12	0.620E-03
2	0.921E-02	13	0.290E-02
3	0.178E-02	15	0.130E-02
4	0.338E-02	16	0.130E-02
5	0.380E-02	17	0.260E-01
6	0.302E-02	18	0.248E-01
7	0.230E-02	19	0.124E-02
8	0.112E-02	20	0.136E-02
9	0.118E-02	21	0.142E-02
10	0.142E-02	25	0.136E-02
11	0.220E-02	26	0.290E-02

Table 12.1. Continued

Line Pwr.	Std. Dev.	Line Pwr.	Std. Dev.
27	0.524E-02	49	0.220E-02
28	0.242E-02	51	0.440E-03
30	0.278E-02	52	0.132E-02
31	0.266E-02	54	0.230E-02
32	0.212E-02	55	0.338E-02
33	0.224E-02	56	0.130E-02
34	0.224E-02	57	0.112E-02
35	0.148E-02	58	0.118E-02
38	0.130E-01	59	0.196E-02
40	0.224E-02	61	0.160E-01
41	0.230E-02	62	0.248E-02
42	0.106E-02	63	0.142E-02
43	0.118E-02	64	0.833E-02
44	0.224E-02	65	0.208E-02
45	0.230E-02	66	0.196E-02
46	0.130E-02	67	0.118E-02
48	0.232E-02	68	0.304E-02

Table 12.1. Continued

Bus Volt	Std. Dev.
3	0.300E-02
4	0.300E-02
5	0.300E-02
7	0.300E-02
8	0.300E-02
11	0.300E-02
13	0.300E-02
24	0.300E-02
25	0.300E-02
30	0.300E-02
36	0.300E-02
37	0.300E-02
40	0.300E-02
41	0.300E-02
55	0.300E-02
56	0.300E-02
58	0.300E-02

B. True State

The state defined to be the true state of the system is shown in Table 12.2. This state was obtained by operating the STATE ESTIMATOR program as a load flow program as described in Section X-A.

C. Measurement Readings

Each of the simulated measurement readings are listed in Table 12.3. These measurements are calculated from the true state of the system and include no intentionally added measurement noise.

D. Optimum Estimate

Using the measurement configuration in Figure 11.1 and the correct model of the system, the STATE ESTIMATOR and SENSITIVITY ANALYSIS programs were run to obtain the optimum state estimate. The data obtained from this optimum estimate provided the following information for the experiment:

1. Accuracy check for the SENSITIVITY ANALYSIS program. The incorrect variance (also referred to as the calculated variance) and the actual variance are found by calculating the diagonal terms of $(F_c' R_c^{-1} F_c)^{-1}$ and $(F_c' R_c^{-1} F_c)^{-1} F_c' R_c^{-1} R R_c^{-1} F_c (F_c' R_c^{-1} F_c)^{-1}$ respectively. These calculations involve finding the inverse of a 115 x 115 matrix without the use of any pivoting for size in the Gaussian elimination process. It is conceivable that round-off errors could affect the

accuracy of such a calculation, so it is highly desirable to determine how much error will be introduced. Using the optimum estimate, the SENSITIVITY ANALYSIS program provides this information in the following manner:

a) Since $R = R_c$, the calculated and actual variances should be the same. This was found to be the case to within seven figures of accuracy. This test was the basis for the decision to use double precision for this calculation since single precision yielded results that agreed only within the first digit.

b) Since $\underline{x}_o = \underline{x}_{oc}$, $\underline{f}(\underline{x}_o) = \underline{f}_c(\underline{x}_{oc})$, $F(\underline{x}_o) = F_c(\underline{x}_{oc})$ and $R = R_c$, the expected error should be equal to zero. The actual data indicated that the value of each expected error for this case was less than 1×10^{-14} pu for voltage magnitudes and 1×10^{-14} radians for phase angles. These errors are several orders of magnitude less than the nominal values and therefore are insignificant.

2. The actual variances of the optimum estimates provide a standard of comparison for the actual variances of those estimates obtained from the incorrect model.

E. Sensitivity to Modeling Errors

The object of this test was to determine how the variance and expected error of the state estimates would be affected by errors in the following parameters:

Table 12.2. True state of the system

All magnitudes are shown in per unit (pu) and all phase angles are shown in radians

Bus	Voltage Magnitude	Phase Angle
1	0.103E 01	0.783E-01
2	0.101E 01	0.119E 00
3	0.104E 01	0.689E-01
4	0.103E 01	0.518E-01
5	0.103E 01	0.239E-01
6	0.102E 01	0.118E-01
7	0.102E 01	0.940E-02
8	0.102E 01	0.940E-02
9	0.102E 01	0.990E-02
10	0.102E 01	0.104E-01
11	0.102E 01	0.108E-01
12	0.102E 01	0.850E-02
13	0.102E 01	0.103E 00
14	0.103E 01	0.474E-01
15	0.103E 01	0.364E-01
16	0.103E 01	0.369E-01
17	0.103E 01	0.280E-01
18	0.102E 01	0.177E-01
19	0.101E 01	0.187E-01
20	0.102E 01	0.443E-01
21	0.102E 01	-0.420E-02

Table 12.2. Continued

Bus	Voltage Magnitude	Phase Angle
22	0.103E 01	-0.110E-02
23	0.103E 01	0.100E-03
24	0.103E 01	0.607E-01
25	0.102E 01	-0.130E-02
26	0.102E 01	-0.330E-02
27	0.102E 01	-0.360E-02
28	0.103E 01	-0.130E-02
29	0.102E 01	-0.100E-03
30	0.103E 01	0.470E-02
31	0.101E 01	-0.680E-02
32	C.100E 01	-0.830E-02
33	0.100E 01	0.220E-02
34	0.101E 01	0.141E-01
35	0.103E 01	0.360E-01
36	0.103E 01	0.407E-01
37	0.103E 01	0.398E-01
38	0.103E 01	-0.110E-02
39	0.103E 01	-0.900E-03
40	0.103E 01	-0.170E-02
41	0.103E 01	-0.270E-02
42	0.103E 01	-0.170E-02
43	0.103E 01	-0.270E-02

Table 12.2. Continued

Bus	Voltage Magnitude	Phase Angle
44	0.103E 01	0.172E-01
45	0.102E 01	-0.300E-02
46	0.100E 01	-0.774E-02
47	0.996E 00	-0.941E-02
48	0.993E 00	-0.815E-02
49	0.989E 00	-0.409E-02
50	0.103E 01	0.800E-03
51	0.103E 01	-0.230E-02
52	0.103E 01	-0.340E-02
53	0.987E 00	-0.233E-03
54	0.101E 01	-0.390E-02
55	0.101E 01	0.104E-01
56	0.103E 01	0.207E-01
57	0.103E 01	0.102E-01
58	0.103E 01	0.0

Table 12.3. Simulated measurement readings

All quantities are shown in per unit (pu)

Head = End of line where line flow measurement is made

Tail = Opposite end from head

Bus	Real Pwr.	React. Pwr.
21	0.108E 00	0.170E-01

Line	Head	Tail	Real Pwr.	React. Pwr.
1	24	36	-0.433E 00	0.592E-02
2	24	4	-0.216E 00	0.143E-01
3	56	24	0.125E 00	0.739E-01
4	25	56	0.226E 00	0.700E-02
5	56	50	-0.300E 00	0.249E-01
6	56	57	-0.172E 00	0.178E-01
7	56	44	-0.578E-01	-0.699E-02
8	56	17	0.234E-01	-0.295E-01
9	56	18	-0.293E-01	-0.317E-01
10	55	56	0.680E-01	-0.592E-02
11	45	29	0.176E 00	0.651E-01
12	55	54	-0.681E-01	0.351E-02
13	58	52	-0.144E 00	0.369E-01
15	19	20	0.516E-01	-0.777E-02

Table 12.3. Continued

Line	Head	Tail	Real Pwr.	React. Pwr.
16	13	14	-0.487E-01	0.573E-01
17	2	3	-0.856E 00	0.107E 01
18	1	3	-0.120E 00	0.784E 00
19	49	48	-0.268E-01	0.672E-01
20	47	48	0.490E-03	-0.631E-01
21	49	53	0.681E-01	-0.587E-01
25	40	39	0.569E-01	0.414E-01
26	58	38	-0.145E 00	-0.771E-01
27	37	58	-0.521E 00	-0.141E 00
28	58	51	-0.688E-01	0.275E-01
30	42	38	0.124E 00	0.124E 00
31	42	43	-0.962E-01	-0.561E-01
32	42	41	-0.223E-01	0.670E-02
33	45	43	0.332E-01	0.804E-01
34	40	41	-0.369E-01	0.109E-01
35	30	28	-0.865E-01	0.265E-02
38	4	37	-0.457E 00	-0.829E-01
40	45	27	-0.317E-01	0.120E-01
41	25	26	-0.393E-01	-0.266E-02
42	32	31	0.180E-01	0.194E-01
43	30	31	-0.329E-01	-0.265E-01
44	32	33	0.332E-01	-0.373E-02
45	34	33	-0.425E-01	0.331E-03

Table 12.3. Continued

Line	Head	Tail	Real Pwr.	React. Pwr.
46	34	35	0.507E-01	0.454E-02
48	23	50	0.291E 00	0.114E 00
49	23	22	-0.129E 00	0.840E-01
51	9	12	-0.352E-01	-0.398E-02
52	9	10	0.298E-01	0.265E-01
54	11	10	-0.491E-01	0.194E-01
55	11	5	0.231E 00	0.952E-02
56	15	17	-0.522E-01	0.321E-01
57	15	16	0.245E-01	0.321E-01
58	15	14	0.339E-01	-0.540E-01
59	8	5	0.156E 00	0.137E-01
61	3	4	-0.967E 00	-0.391E 00
62	45	46	-0.425E-01	-0.902E-01
63	47	46	0.463E-01	0.607E-01
64	4	5	-0.723E 00	-0.328E-01
65	7	5	0.177E 00	-0.560E-02
66	5	6	-0.159E 00	-0.407E-02
67	19	18	0.103E-01	0.294E-01
68	30	36	0.324E 00	0.830E-01

Table 12.3. Continued

Bus	Volt Mag.	Bus	Volt Mag.
3	0.104E 01	4	0.103E 01
5	0.103E 01	7	0.102E 01
8	0.102E 01	11	0.102E 01
13	0.102E 01	24	0.103E 01
25	0.102E 01	30	0.103E 01
36	0.103E 01	37	0.103E 01
40	0.103E 01	41	0.103E 01
55	0.101E 01	56	0.103E 01
58	0.103E 01		

1. Line capacitance
2. Line inductance
3. Line resistance
4. Variance of measurement errors
5. Transformer tap settings

All data was obtained by adding a known error to the set of parameters in question and then comparing the results with those of the correct model.

In all cases, the STATE ESTIMATOR program was initialized from a semi-flat start; i.e., all voltage magnitudes = 1.0 and all phase angles = 0.0 with the exception of bus #1 where the voltage and phase angle were set equal to 1.04 and 0.08 respectively. This starting point was used to determine if the program would still converge from a point some distance from the final answer with various modeling errors present. In all cases the program converged in three iterations, which was the same number required for the correct model.

Tables 12.4 through 12.23 on pages 104 through 113 list the maximum expected errors and actual variances along with the corresponding optimum variances (based on the correct model) for each set of parameters. The data in these tables indicates the relative effects of each parameter but does not necessarily indicate whether the errors shown will be acceptable. This will naturally depend on how the data is to be used, and some intended applications will certainly require greater accuracy than others. If this data is intended for tracking voltage magnitudes or for monitoring phase angle differences for stability purposes, most of these errors are probably acceptable. However, if the state estimates are to be used for on-line contingency studies or for calculating line flows where no measurements are available, better accuracy may be required; i.e., a more accurate model will be necessary.

The question of accuracy for contingency studies and line flow calculations is quite difficult to answer without actually calculating each of the power levels desired. The reason for this is that the calculations may involve small differences of relatively large numbers, so that an error in the state which appears to be slight can have a large effect on the calculated power. A few example calculations were chosen for each type of parameter variation, and the results are shown in Table 24 on page 114. This table indicates the error in both real and reactive line flows for lines 23, 29, and 47. The errors in real and reactive power are first expressed as a percentage of the actual values, and then both are expressed as a percentage of the maximum MVA rating of the line (i.e., $(g_{calc.}-g_{act.})/g_{rate} \times 100\%$). The percentage error based on the rating is included because one of the main uses for this data is to check that line ratings are not exceeded, in which case large percentage errors based on light loading conditions are not too important. Tables 25, 26, and 27 on pages 115 through 117 also list the rated and actual MVA, the expected errors in the estimated voltages and phase angles for each line, and the actual voltages and phase angles for each line, respectively. Further comments on these results will be made in the following discussions pertaining to each type of modeling error.

It should also be emphasized that the 10% errors for resistance, inductance, and capacitance are intended to indicate the relative effects of these parameters and are probably 3 to 4 times the normal error for inductance and capacitance (see reference 15 and Section 9-A). Thus the errors shown for these two parameters are likely to be somewhat excessive.

1. Line capacitance

In this test, the capacitance of each line was increased by 10%, and the state estimates were calculated using the same measurement values as for the optimum estimates. Tables 12.4 and 12.5 on page 104 list the results for the three largest variations in expected error in voltage magnitudes and phase angles, while Tables 12.6 and 12.7 on page 105 list the largest percentage deviations from the optimum variance for voltages and phase angles, respectively. These capacitance errors appear to have little effect on the variance of the estimates, and most of the errors in the unmeasured power calculations of Table 12.24 on page 114 tend to be small, especially when expressed as a percentage of the maximum rating.

2. Line inductance

In a test similar to that for capacitance, the inductance value of each line was increased by 10%, and the effects on the state estimates are listed in Tables 12.8 through 12.11 on pages 106 and 107. These tables indicate that the errors in inductance have a greater effect on the estimates than the errors in capacitance and that the deviations from the optimum variance are more significant. Some of the power calculation errors in Table 12.24 are quite large compared to the actual values but tend to be relatively small when referenced to the line ratings.

3. Line resistance

As was done for line capacitance and inductance, the resistance of each line was increased by 10% to determine the effect on the state estimates. Table 9.1, which is based on data from reference 15, indicates a typical error of 8.3% for this parameter, so the 10% figure seems quite representative of what might be expected in practice. Tables 12.12 through 12.15 on pages 108 and 109 indicate that these resistance errors have little effect on the variance of the estimates. Table 12.24 indicates that most of the power calculation errors tend to be small, especially in comparison with the maximum line ratings.

4. Variance of measurement errors

As pointed out in Section IX-C, large errors may be present in the values used for the variance, σ^2, of the measurement errors (i.e., assuming that $\sigma^2 = 4\%$ when it is actually 9%, for example). Much of this data will probably be little more than a rough approximation, so it was assumed that the standard deviation, σ, of the measurement errors could vary by $\pm 50\%$ (i.e., if $\sigma = 2\%$ it can vary from 1% to 3% in absolute value). It is conceivable that even this may be an optimistic figure, but it should be sufficiently large to give an indication of the sensitivity to this parameter.

If all terms in the measurement error covariance matrix, R, are increased by the same percentage, there will be a tendency to increase the weight of each measurement by the same relative amount. To avoid this the σ of every other measurement was increased by 50% and the remaining ones were decreased by 50%. For the power measurements this resulted in a 50% increase in the σ of all the real power levels and a 50% decrease in the σ of all the reactive power levels. The most sensitive states are listed in Tables 12.16 through 12.19 on page 110 and 111 along with the expected error, optimum variance, and actual variance.

One very encouraging result is that the expected errors of the estimates are a few orders of magnitude less than those obtained for the errors in the electrical parameters. This at least indicates that when accurate measurements are used, large errors in the relative weighting of these measurements have a relatively small effect on the expected error of the state estimates. However, this may not be the case when accurate and inaccurate measurements are mixed and the wrong weighting factors are used. Table 12.24 indicates that the effect of these estimate errors on the calculated line flows will also be quite small.

As might be expected, some rather large deviations appear between the optimum and actual variances of these state estimates. This should be regarded as a significant problem, since the variance is a measure of how the errors may deviate from the average value, and many of the actual variances shown here are considerably larger than the optimum value. This test points out the need for examining the actual and optimum variance, since the expected error alone would give little indication of the increase in the dispersion of these estimates.

5. Transformer tap settings

This system contains four TCUL transformers which are represented by lines 27, 46, 64, and 68 in Figure 11.1. To determine the effects of an incorrect tap ratio, the nominal ratios shown in Table 11.3 were arbitrarily changed to the following values, all of which are within the allowable range of positions for this device:

Tap 27 = 0.85

Tap 46 = 1.15

Tap 64 = 1.15

Tap 68 = 0.85

The results shown in Tables 12.20 through 12.23 on page 112 and 113 indicate that the effects of this error are quite significant. Bus 35 has an expected error in voltage magnitude of approximately 25% while buses 32 and 33 have errors of approximately 11% each. Some of the expected errors shown here will undoubtedly be intolerable for many intended uses of the estimates, so it appears it will be necessary to monitor these tap ratios in an actual application. This assumption is easily justified by examining the calculated power level errors in Table 12.24.

The results also indicate a very significant change in the variances of the estimates. It is interesting to note from Table 12.20 that the voltage estimate at bus #35 has a variance considerably less than the optimum value obtained from the correct model, even though the expected error has increased from 0% to 25%. This example demonstrates how a biased estimate can have a lower variance than the unbiased estimate based on the correct model.

F. Sensitivity to Large Measurement Errors

As discussed in Section IX-D, the measurements may contain errors considerably larger than those normally encountered and yet small enough that they will be difficult to detect. It is certainly of interest to determine how such errors will affect the state estimates, so an attempt has been made here to at least give an indication of this. To conduct this study, the following 16 measurements were arbitrarily given a 10% error:

Bus 21: Real Power = +10%, Reactive Power = -10%

Lines 3, 10, 20, 40, and 58: Real Power = +10%, Reactive Power = -10%

Buses 4 and 11: Voltage Magnitude = -10%

Buses 36 and 58: Voltage Magnitude = +10%

The results of this test, listed in Tables 12.28 through 12.31 on page 118 and 119, show that both the expected errors and deviations from the optimum variance are relatively small compared to the results for the various modeling errors. It is of interest to compare the voltage estimates at buses 4, 11, 36, and 58 from the STATE ESTIMATOR program with the measured values and true values:

Voltage	True Value	Measured	Estimated
4	1.033 pu	0.927 pu	1.031 pu
11	1.025	0.918	1.021
36	1.029	1.130	1.031
58	1.029	1.130	1.031

Note that the estimates show a significant improvement over the measured values due to the combination of accurate line flow measurements with the inaccurate voltage measurements.

This data demonstrates that the STATE ESTIMATOR program does have the ability to correct for measurement errors in certain cases, but it would probably be unwise to draw any general conclusions about this characteristic until more extensive testing is performed.

Table 12.4. Buses with maximum expected errors in voltage magnitude with +10% errors in capacitance

Bus	Exp. Err.	Opt. Var.	Act. Var.
8V	0.444E-02	C.668E-C6	0.673E-06
8A	0.356E-03	C.426E-06	0.426E-06
7V	C.444E-02	C.664E-06	0.669E-06
7A	0.356E-03	C.421E-C6	0.422E-06
12V	C.444E-02	0.677E-06	0.682E-06
12A	C.365E-03	C.432E-C6	0.433E-06

Table 12.5. Buses with maximum expected errors in phase angle with +10% errors in capacitance

Bus	Exp. Err.	Opt. Var.	Act. Var.
12V	C.444E-02	C.677E-06	0.682E-06
12A	C.365E-C3	C.432E-06	0.433E-06
7V	0.444E-02	C.664E-C6	0.669E-06
7A	C.356E-C3	C.421E-C6	0.422E-06
8V	C.444E-02	C.668E-06	0.673E-06
8A	C.356E-03	C.426E-06	0.426E-06

Table 12.6. Buses with maximum change in variance of voltage magnitude with +10% errors in capacitance

Bus	Exp. Err.	Opt. Var.	Act. Var.
5V	0.443E-02	C.630E-06	0.636E-06
5A	0.234E-03	C.391E-06	0.392E-06
6V	0.444E-C2	C.661E-06	0.666E-06
6A	C.340E-03	C.415E-06	0.416E-06
7V	C.444E-C2	C.664E-06	0.669E-06
7A	0.356E-03	C.421E-C6	0.422E-06

Table 12.7. Buses with maximum change in variance of phase angle with +10% errors in capacitance

Bus	Exp. Err.	Opt. Var.	Act. Var.
3V	-0.153E-02	C.647E-06	0.646E-06
3A	C.2C1E-03	C.377E-06	0.379E-06
4V	-C.155E-02	0.581E-06	0.580E-06
4A	C.151E-03	C.292E-06	0.293E-06
53V	-0.150E-02	C.114E-05	0.114E-05
53A	-0.788E-06	C.302E-06	0.304E-06

Table 12.8. Buses with maximum expected errors in voltage magnitude with +10% errors in inductance

Bus	Exp. Err.	Opt. Var.	Act. Var.
13V	-C.639E-02	C.174E-05	0.188E-05
13A	C.710E-02	C.192E-C5	0.224E-05
53V	-0.554E-02	C.114E-05	0.125E-05
53A	-C.1C9E-02	C.302E-06	0.367E-06
7V	0.538E-02	C.664E-06	0.696E-06
7A	0.142E-02	C.421E-C6	0.508E-06

Table 12.9. Buses with maximum expected errors in phase angle with +10% errors in inductance

Bus	Exp. Err.	Opt. Var.	Act. Var.
2V	-C.382E-02	C.345E-C5	0.416E-05
2A	C.120E-01	C.250E-C5	0.305E-05
1V	-0.102E-02	C.474E-05	0.573E-05
1A	C.788E-02	C.336E-C5	0.406E-05
13V	-0.639E-02	C.174E-05	0.188E-05
13A	0.710E-02	C.192E-05	0.224E-05

Table 12.10. Buses with maximum change in variance of voltage magnitude with +10% errors in inductance

Bus	Exp. Err.	Opt. Var.	Act. Var.
1V	-C.102E-C2	C.474E-05	0.573E-05
1A	C.788E-02	C.336E-C5	0.406E-05
2V	-0.382E-02	0.345E-05	0.416E-05
2A	0.120E-01	C.250E-C5	0.305E-05
35V	-C.298E-02	C.283E-05	0.326E-05
35A	C.314E-02	C.253E-C5	0.304E-05

Table 12.11. Buses with maximum change in variance of phase angle with +10% errors in inductance

Bus	Exp. Err.	Opt. Var.	Act. Var.
2V	-0.382E-02	C.345E-05	0.416E-05
2A	0.120E-01	C.250E-C5	0.305E-05
37V	-C.637E-03	C.647E-06	0.670E-06
37A	C.4C8E-02	C.165E-06	0.200E-06
54V	-C.202F-02	C.843E-C6	0.899E-06
54A	-C.423E-03	0.827E-06	0.101E-05

Table 12.12. Buses with maximum expected errors in voltage magnitude with +10% errors in resistance

Bus	Exp. Err.	Opt. Var.	Act. Var.
5V	0.447E-02	C.630E-06	0.636E-06
5A	0.217E-03	C.391E-06	0.393E-06
8V	0.420E-02	C.668E-06	0.674E-06
8A	0.359E-C3	C.426E-06	0.427E-06
11V	0.413E-02	C.672E-C6	0.678E-06
11A	C.337E-03	0.430E-06	0.432E-06

Table 12.13. Buses with maximum expected errors in phase angle with +10% errors in resistance

Bus	Exp. Err.	Opt. Var.	Act. Var.
13V	0.198E-02	C.174E-05	0.180E-05
13A	C.398E-02	C.192E-05	0.203E-05
20V	-C.500E-03	C.154E-C5	0.158E-05
20A	C.129E-02	C.144E-05	0.148E-05
14V	-C.276E-C3	C.954E-C6	0.960E-06
14A	0.123E-02	C.895E-C6	0.909E-06

Table 12.14. Buses with maximum change in variance of voltage magnitude with +10% errors in resistance

Bus	Exp. Err.	Opt. Var.	Act. Var.
13V	0.198E-02	0.174E-05	0.180E-05
13A	0.398E-02	0.192E-05	0.203E-05
20V	-0.500E-03	0.154E-05	0.158E-05
20A	0.129E-02	0.144E-05	0.148E-05
34V	-0.164E-02	0.240E-05	0.245E-05
34A	0.812E-03	0.221E-05	0.227E-05

Table 12.15. Buses with maximum change in variance of phase angle with +10% errors in resistance

Bus	Exp. Err.	Opt. Var.	Act. Var.
13V	0.198E-02	0.174E-05	0.180E-05
13A	0.398E-02	0.192E-05	0.203E-05
51V	-0.162E-02	0.768E-06	0.768E-06
51A	-0.381E-04	0.568E-08	0.588E-08
20V	-0.500E-03	0.154E-05	0.158E-05
20A	0.129E-02	0.144E-05	0.148E-05

Table 12.16. Buses with maximum expected errors in voltage magnitude with ±50% errors in standard deviation of measurements

Bus	Exp. Err.	Opt. Var.	Act. Var.
29V	-0.282E-05	C.778E-06	0.138E-05
29A	-0.633E-06	C.960E-08	0.959E-08
16V	C.246E-05	0.910E-06	0.162E-05
16A	-0.168E-05	C.858E-C6	0.864E-06
13V	0.213E-05	0.174E-05	0.232E-05
13A	-C.259E-C5	C.192E-05	0.194E-05

Table 12.17. Buses with maximum expected errors in phase angle with ±50% errors in standard deviation of measurements

Bus	Exp. Err.	Opt. Var.	Act. Var.
13V	0.213E-05	C.174E-05	0.232E-05
13A	-0.259E-05	0.192E-05	0.194E-05
35V	C.760E-06	C.283E-05	0.340E-05
35A	-C.240E-05	0.253E-C5	0.254E-05
34V	0.609E-07	0.240E-05	0.294E-05
34A	-C.236E-05	C.221E-C5	0.221E-05

Table 12.18. Buses with maximum change in variance of voltage magnitude with $\pm$50% errors in standard deviation of measurements

Bus	Exp. Err.	Opt. Var.	Act. Var.
56V	0.135E-05	0.826E-06	0.153E-05
56A	-0.159E-05	0.761E-06	0.767E-06
44V	0.182E-05	0.856E-06	0.157E-05
44A	-0.144E-05	0.786E-06	0.793E-06
55V	0.151E-05	0.823E-06	0.150E-05
55A	-0.167E-05	0.808E-06	0.815E-06

Table 12.19. Buses with maximum change in variance of phase angle with $\pm$50% errors in standard deviation of measurements

Bus	Exp. Err.	Opt. Var.	Act. Var.
3V	-0.319E-06	0.647E-06	0.102E-05
3A	-0.196E-05	0.377E-06	0.384E-06
37V	-0.195E-05	0.647E-06	0.108E-05
37A	-0.356E-06	0.165E-06	0.168E-06
4V	0.116E-05	0.581E-06	0.965E-06
4A	-0.134E-05	0.292E-06	0.296E-06

Table 12.20. Buses with maximum expected errors in voltage magnitude with ±15% errors in TCUL tap settings

Bus	Exp. Err.	Opt. Var.	Act. Var.
35V	-0.250E 00	0.283E-05	0.228E-05
35A	0.866E-02	0.253E-05	0.364E-05
32V	-0.111E 00	0.139E-05	0.137E-05
32A	-0.282E-02	0.118E-05	0.149E-05
33V	-0.110E 00	0.192E-05	0.204E-05
33A	-0.845E-04	0.172E-05	0.234E-05

Table 12.21. Buses with maximum expected errors in phase angle with ±15% errors in TCUL tap settings

Bus	Exp. Err.	Opt. Var.	Act. Var.
21V	0.598E-01	0.915E-06	0.951E-06
21A	0.137E-01	0.834E-06	0.770E-06
54V	0.576E-01	0.843E-06	0.880E-06
54A	0.137E-01	0.827E-06	0.765E-06
26V	0.592E-01	0.976E-06	0.101E-05
26A	0.136E-01	0.909E-06	0.831E-06

Table 12.22. Buses with maximum change in variance of voltage magnitude with $\pm$15% errors in TCUL tap settings

Bus	Exp. Err.	Opt. Var.	Act. Var.
35V	-0.250E 00	C.283E-05	0.228E-05
35A	C.866E-02	0.253E-05	0.364E-05
30V	-C.107E 00	C.874E-06	0.722E-06
30A	C.463E-03	C.723E-06	0.770E-06
28V	-0.1C8E 00	0.889E-06	0.741E-06
28A	-C.103E-02	C.735E-06	0.789E-06

Table 12.23. Buses with maximum change in variance of phase angle $\pm$15% errors in TCUL tap settings

Bus	Exp. Err.	Opt. Var.	Act. Var.
53V	-0.102E 00	0.114E-05	0.111E-05
53A	C.754E-04	C.302E-06	0.458E-06
49V	-0.102E 00	C.113E-05	0.109E-05
49A	-0.867E-03	C.298E-06	0.450E-06
48V	-0.102E 00	C.111E-05	0.107E-05
48A	-C.185E-02	C.288E-06	0.435E-06

Table 12.24. Examples of % errors in calculated values of unmeasured line flow power levels

Parameter Errors in Other Lines	Line Under Study	% Error in Calculated Line Flow Power			
		% of $g_{act.}$	% of $h_{act.}$	% of g_{rate}	% of h_{rate}
+10% Capacitance	23	3.155	1.189	0.419	0.710
	29	1.505	-14.444	0.253	0.421
	47	1.799	-4.592	0.522	1.031
+10% Inductance	23	44.420	13.680	5.899	8.169
	29	10.623	83.635	1.789	-2.440
	47	0.894	17.975	0.259	-4.034
+10% Resistance	23	-19.257	3.862	-2.557	2.306
	29	-1.009	8.458	-0.170	-0.247
	47	-3.592	8.922	-1.042	-2.002
±50% Measurement Error Standard Deviation	23	-0.005	0.020	-0.001	0.012
	29	-0.017	-0.201	-0.003	0.006
	47	-0.005	-0.048	-0.001	0.011
±15% Tap Ratio Error	23	1537.0	604.12	203.84	360.75
	29	882.51	10153	148.61	302.74
	47	-1200.0	3130.0	347.69	701.89

Table 12.25. Rated and actual MVA levels for lines 23, 29, and 47

Line	Max. Rated MVA	Act. Real MVA	Act. Reactive MVA
23	30	4.0	17.9
29	57	9.6	-1.7
47	57	16.5	-12.8

Table 12.26. Expected errors in estimated values for those states associated with lines 23, 29, and 47

Parameter Errors in Other Lines	Line Under Study	Expected Errors in State Estimates			
		ΔV_{head} (pu)	$\Delta\delta_{head}$ (rad.)	ΔV_{tail} (pu)	$\Delta\delta_{tail}$ (rad.)
+10% Capacitance	23	-0.001505	-0.000001	-0.001191	0.000019
	29	-0.001439	0.0	-0.001231	0.000051
	47	-0.001440	-0.000006	-0.001234	0.000030
+10% Inductance	23	-0.005538	-0.001091	-0.002025	-0.000423
	29	-0.001479	0.0	-0.002154	0.001077
	47	-0.001399	-0.000206	-0.002073	0.000116
+10% Resistance	23	-0.001897	0.001119	-0.001488	-0.000014
	29	-0.001535	0.0	-0.001658	-0.000007
	47	-0.001617	-0.000038	-0.002031	-0.000062
±50% Measurement Error Standard Deviation	23	-0.000002	0.000001	0.000002	-0.000001
	29	-0.000002	0.0	0.000001	-0.000001
	47	-0.000002	0.0	0.000001	-0.000002
±15% Tap Ratio Error	23	-0.102404	0.000075	0.057638	0.013651
	29	-0.097845	0.0	0.059491	0.012174
	47	-0.097843	-0.000509	0.059639	0.013176

Table 12.27. True values for those states associated with lines 23, 29, and 47

Line	V_{head}(pu)	δ_{head}(rad.)	V_{tail}(pu)	δ_{tail}(rad.)
23	0.987	-0.00023	1.007	-0.00390
29	1.029	0.0	1.031	0.01020
47	1.029	-0.00230	1.028	0.00080

Table 12.28. Buses with maximum expected errors in voltage magnitude with $\pm 10\%$ measurement errors

Bus	Exp. Err.	Opt. Var.	Act. Var.
55V	C.117E-04	C.823E-06	0.826E-06
55A	0.672E-C5	C.808E-C6	0.8C8E-06
21V	0.110E-04	C.915E-06	0.919E-06
21A	C.149E-04	C.834E-06	0.835E-06
44V	C.110E-04	C.856E-C6	0.859E-06
44A	0.127E-C4	C.786E-06	0.786E-06

Table 12.29. Buses with maximum expected errors in phase angle with $\pm 10\%$ measurement errors

Bus	Exp. Err.	Opt. Var.	Act. Var.
21V	C.110E-04	C.915E-06	0.919E-06
21A	0.149E-04	C.834E-06	0.835E-06
25V	0.102E-04	C.957E-C6	0.960E-06
25A	0.139E-04	0.892E-06	0.893E-06
26V	C.1C8E-C4	C.976E-06	0.979E-06
26A	0.139E-04	C.909E-06	0.911E-06

Table 12.30. Buses with maximum change in variance of voltage magnitude with ±10% measurement errors

Bus	Exp. Err.	Opt. Var.	Act. Var.
34V	-C.400E-05	0.240E-C5	0.238E-05
34A	-C.1C9E-05	C.221E-05	0.219E-05
35V	-C.533E-05	C.283E-05	0.282E-05
35A	-C.110E-05	C.253E-C5	0.251E-05
33V	-C.421E-05	C.192E-05	0.191E-05
33A	-C.944E-C6	0.172E-05	0.170E-05

Table 12.31. Buses with maximum change in variance of phase angle with ±10% measurement errors

Bus	Exp. Err.	Opt. Var.	Act. Var.
35V	-C.533E-05	C.283E-C5	0.282E-05
35A	-C.110E-05	0.253E-05	0.251E-05
48V	-C.595E-05	C.111E-C5	0.111E-05
48A	0.376E-07	C.288E-06	0.285E-06
47V	-C.585E-C5	C.110E-05	0.110E-05
47A	-C.357E-06	C.283E-06	0.280E-06

XIII. CONCLUSIONS AND SUGGESTIONS FOR FURTHER RESEARCH

As shown in previous studies, state estimation programs can play an important role in monitoring electrical power systems due to the large amounts of information that they can provide. However, in order to determine the accuracy of the results that can be obtained, it is important to study the sensitivity of these estimators to errors in the system model. A method for evaluating the sensitivity of weighted least-squares estimators has been developed here, and it is based on the following criteria:

1. The expected errors is the estimates
2. The optimum and actual variance of the estimates
3. The effects of erroneous estimates upon subsequent power calculations

The first two items evaluate the very properties of the weighted least-squares estimator that make it attractive, namely that if the model is correct it should provide the minimum variance among all unbiased estimates. These two items also have a natural interpretation from an engineering standpoint, since they indicate the average errors in the estimates and how the individual errors will be dispersed about this average value. The last item is of particular interest since the calculation of unmeasured power levels is undoubtedly one of the most important pieces of data that can be found from the state estimates.

To uncover some of the practical aspects of these sensitivity studies, the method proposed here was applied to a network model based on the Iowa Power and Light Company's Central Division. This simulation produced a number of interesting results and also pointed to additional areas where further research is needed.

A. Conclusions

The following are some of the more important conclusions that were drawn from this study:

1. The proposed sensitivity analysis method produces results that are meaningful from both a mathematical and a physical standpoint. The criteria are based on properties of the estimator that are well defined, and the experimental results indicate how the estimates and subsequent power calculations will be affected by each type of modeling error.
2. When the transmission line parameters were varied, Tables 12.4 through 12.15 indicate that +10% errors in inductance cause somewhat larger errors in the state estimates then corresponding errors in resistance and capacitance. The effect of these parameter errors on the variance of the estimates is slight. Table 12.24 indicates that these errors in the estimates can cause large errors in subsequent power calculations, especially in the case of line inductance. The most serious errors tend to occur in lightly loaded lines however, and most errors are relatively small when compared with the line ratings.

3. Tables 12.16 through 12.19 indicate that variations of $\pm 50\%$ in the value of the standard deviation of the measurement errors (i.e., assuming $\sigma = 3\%$ instead of 2% for example) have a very small effect on the average error in the state estimates, but that the variance may be considerably larger than the optimum value. This is an important result since it indicates how the individual estimate errors will tend to increase when measurement errors are present. Table 12.24 indicates that the average estimate errors will have little effect on the power calculations.
4. Tables 12.20 through 12.21 indicate that $\pm 15\%$ errors in TCUL transformer tap settings can cause large errors in both the average error and variance of the state estimates. Table 12.24 indicates that the resulting errors in the power calculations make these results virtually meaningless. The obvious conclusion here is that tap positions should be monitored and made available to the state estimation program.
5. Tables 12.28 through 12.31 demonstrate that the state estimation program does have the ability to suppress the effects of measurement errors within a $\pm 10\%$ range. Only a small amount of data was obtained here however, and it would probably be unwise to draw any general conclusions without further testing.

B. Areas for Further Research

The experimental portion of this study indicated a number of areas where further research is needed, and many of these could be of great practical value. The following is a list of some of the more significant areas encountered:

1. Due to the length of the exact algorithm, no effort was made to evaluate the Kalman filter approach discussed in Appendix A. If the computation time could be reduced to a reasonable level, there may be some advantages to this approach since it has the capability of decreasing the effects of measurement errors that can be characterized by white noise.
2. Many practical experiments remain to be performed such as, 1) evaluating the effects of open lines as discussed in Appendix B; 2) testing different measurement configurations; 3) further examination of the benefits of redundant measurements and the effect of improper weighting when measurement errors are present, and 4) comparing results taken at different loading conditions.
3. A number of improvements could probably be made in the computer programs used in this study, especially in the area of adjusting the STATE ESTIMATOR program to changes in the system model. This is of particular importance for changes in TCUL transformer tap settings, since the results of this study indicate that these should be accounted for, which means that the parameters of the program must be changed on-line.
4. This study indicates how the average errors in the estimates would affect subsequent power calculations, but it gives no

indication of how these errors in the calculations will be dispersed. In effect what is needed here is to determine how the errors in the power calculations will be distributed, just as we have already shown how the errors in the state estimates will be distributed. Since the power levels are nonlinear functions of the state, some approximations will be involved, but it should be possible to approach the problem in the following manner,

$\underline{x}$ = true state

$\hat{\underline{X}}$ = optimum state estimate

$\hat{\underline{X}}_c$ = state estimate with modeling errors present

$\underline{h}(\underline{x})$ = power levels to be calculated

$H(\underline{x})$ = Jacobian matrix of $\underline{h}(\underline{x})$

$\underline{\mu}_c = E(\hat{\underline{X}}_c)$

P = covariance of $\hat{\underline{X}}$

P_a = actual covariance of $\hat{\underline{X}}_c$

Using a Taylor's series,

$$^{1}\underline{h}(\hat{\underline{X}}_c) \simeq \underline{h}(\underline{x}) + H(\underline{x})\,[\hat{\underline{X}}_c - \underline{x}] \tag{13.1}$$

and since $\hat{\underline{X}}_c \sim N(\underline{\mu}_c, P_a)$ it follows that

$$\underline{h}(\hat{\underline{X}}_c) \sim N(\underline{h}(\underline{x}) + H(\underline{x})(\underline{\mu}_c - \underline{x}),\ H(\underline{x})P_aH'(\underline{x})) \tag{13.2}$$

---

[1] $\underline{h}(\hat{\underline{X}}_c)$ and $\underline{h}(\hat{\underline{X}})$ are random variables.

In a similar fashion for the optimum estimate, $\underline{\hat{X}}$, we have,

$$\underline{h}(\underline{\hat{X}}) \sim N(\underline{h}(\underline{x}),\ H(\underline{x})PH'(\underline{x})) \tag{13.3}$$

The results of Equation 13.2 and 13.3 can then be compared to determine how the distribution of the calculated power levels will be affected by state estimate errors.

XIV. LITERATURE CITED

1. P. M. Anderson, "Analysis of Faulted Power Systems," Unpublished class notes for EE 541. Ames, Iowa, Electrical Engineering Department, Iowa State University of Science and Technology. 1968.

2. A. S. Debs and R. E. Larson, "A Dynamic Estimator for Tracking the State of a Power System," IEEE Trans. Power Apparatus and Systems, Vol. PAS-89, No. 7, p. 1670, September/October, 1970.

3. A. S. Debs and W. H. Litzenberger, "Experimental Evaluation of Tracking State Estimation," presented at the PICA Conference, Boston, Mass., May 23-26, 1971.

4. E. E. Fetzer, "Observability in the State Estimation of Power Systems," Unpublished Ph.D. thesis. Ames, Iowa, Library, Iowa State University of Science and Technology. 1972.

5. L. P. Hajdu, "Modeling of Meter Errors for State Estimation," Unpublished company memorandum. Palo Alto, California, Systems Control, Inc., April 3, 1970.

6. L. P. Hajdu, D. W. Bree, and A. N. Brooks, "On-line Monitoring of Power System Security," presented at the IEEE Summer Power Meeting and EHV Conference, Los Angeles, California, July 12-17, 1970.

7. E. Hnyilicza, "A Power System State Estimator with Applications," PSEG Report No. 12, Department of Electrical Engineering, M.I.T., Cambridge, Mass., April, 1969.

8. R. V. Hogg and A. T. Craig, <u>Introduction to Mathematical Statistics</u>, Macmillan Co., London, 1970.

9. IBM Research Division, "Technical Feasibility of On-Line Load Flow Programs," Power System Computer Feasibility Study, Vol. I, Section IV, San Jose, California, Author. December, 1968.

10. R. E. Larson, W. F. Tinney, and J. Peschon, "State Estimation in Electric Power Systems, Part I: Theory and Feasibility," IEEE Trans. Power Apparatus and Systems, Vol. PAS-89, No. 1, p. 345, January, 1970.

11. R. E. Larson, W. F. Tinney, L. P. Hajdu, and D. S. Piercy, "State Estimation in Electric Power Systems, Part II: Implementation and Applications," IEEE Trans. Power Apparatus and Systems, Vol. PAS-89, No. 1, p. 353, January, 1970.

12. C. T. Leondes, Advances in Control Systems, Theory and Applications, Vol. 3, Academic Press, Inc., New York, 1966.

13. R. D. Masiello and F. C. Schweppe, "A Tracking State Estimator," IEEE Trans. Power Apparatus and Systems, Vol. PAS-90, No. 3, p. 1025, May/June, 1971.

14. H. M. Merrill and F. C. Schweppe, "Bad Data Suppression in Power System Static State Estimation," presented at the IEEE Summer Meeting and International Symposium on High Power Testing, Portland, Oregon, July 18-23, 1971.

15. C. W. Minard, R. B. Gow, W. A. Wolfe, and E. A. Swanson," Staged Fault Tests of Relaying and Stability on Kansas-Nebraska 270 Mile 154 Kv Interconnection," AIEE Trans., Vol. 62, p. 358, 1943.

16. N. E. Nahi, Estimation Theory and Applications, John Wiley & Sons, Inc., New York, 1969.

17. S. Perlis, Theory of Matrices, Addison-Wesley Publishing Co., Inc., Reading, Mass., 1952.

18. C. R. Rao, Linear Statistical Inference and Its Applications, John Wiley & Sons, Inc., New York, 1965.

19. D. B. Rom, "Real Power Redistribution After System Outages; Error Analysis," PSEG Report No. 7, Department of Electrical Engineering, M.I.T., Cambridge, Mass., August, 1968.

20. F. C. Schweppe and J. Wildes, "Power System Static State Estimation, Part I: Exact Model," IEEE Trans. Power Apparatus and Systems, Vol. PAS-89, No. 1, p. 120, January, 1970.

21. F. C. Schweppe and D. Rom, "Power System Static State Estimation, Part II: Approximate Model," IEEE Trans. PA&S, Vol. PAS-89, No. 1, p. 125, January, 1970.

22. F. C. Schweppe, "Power System Static State Estimation: Implementation," IEEE Trans. Power Apparatus and Systems, Vol. PAS-89, No. 1, p. 130, January, 1970.

23. F. C. Schweppe and R. Masiello, "Tracking Static State Estimation for Electric Power Systems," PSEG Report No. 27, Department of Electrical Engineering, M.I.T., Cambridge, Mass., February, 1971.

24. O. J. M. Smith, "Power System State Estimation," IEEE Trans. Power Apparatus and Systems, Vol. PAS-89, No. 3, p. 363, March, 1970.

25. G. W. Stagg and A. H. El-Abiad, Computer Methods in Power System Analysis, McGraw-Hill Book Co., New York, 1968.

26. G. W. Stagg, J. F. Dopazo, O. A. Klitin, and L. S. Van Slyck, "Techniques for the Real-time Monitoring of Power Systems Operations," IEEE Trans. Power Apparatus and Systems, Vol. PAS-89, No. 4, p. 545, April, 1970.

27. ___________, "State Calculation of Power Systems from Line Flow Measurements," IEEE Trans. Power Apparatus and Systems, Vol. PAS-89, No. 7, p. 1698, September/October, 1970.

28. W. D. Stevenson, Elements of Power System Analysis, McGraw-Hill Book Co., New York, 1962.

29. L. K. Timothy and B. E. Bona, State Space Analysis: An Introduction, McGraw-Hill Book Co., New York, 1968.

30. J. M. Wildes, "A Static State Estimator for a Power System Network," PSEG Report No. 8, Department of Electrical Engineering, M.I.T., Cambridge, Mass., August, 1968.

XV. ACKNOWLEDGMENTS

The author would like to express his appreciation to Dr. C. J. Herget and Dr. R. G. Brown of Iowa State for their guidance and suggestions on this project and to Mr. Paul McMullin of Iowa Power and Light Company who supplied much of the necessary data. Financial assistance for this research was provided jointly by Project Themis and the Affiliate Research Program in Electric Power at Iowa State University.

XVI. APPENDIX A: ALTERNATE KALMAN FILTERING APPROACH

As pointed out in Section II, the Kalman filtering approach as proposed by Systems Control, Incorporated appears to have some important disadvantages. One of the main problems with this method is the question as to how bias errors should be accounted for. Any attempt to model these errors as white noise is probably nothing more than nonsense, and including them as extra states to be estimated may drastically increase the dimension of the problem. However, we must recognize that both bias errors and white noise type errors may be present in our measurements, so that past measurements should be of some use in averaging out the white noise component. For this reason the following alternate Kalman filtering approach should prove to be of considerable interest. In this method, the statistics of the bias errors and white noise errors are accounted for separately, and no extra state variables are required since no attempt is made to estimate the bias errors. The development here is based on a method proposed earlier by S. F. Schmidt[1] for the study of navigation problems. It will be noted that this technique requires more information about the system, and the equations are somewhat more involved than in the weighted least-squares approach. Therefore the actual on-line implementation of this estimator may not be very practical using present day system information and computation

---

[1] (12, p. 335).

equipment. The technique is certainly of academic interest however, and since it should provide an optimum result, it may be of some use as a standard for comparing different estimators in off-line studies.

First of all, it is necessary to develop a model that describes the dynamic behavior of the power system. The best model would probably involve some set of discontinuous nonlinear differential equations with time varying coefficients. However, we do not begin to have enough information about the behavior of the loads to establish an exact model of this type. Therefore, for lack of better information and since our measurements are made at discrete points in time, we will model the system as the following discrete process,

$$\underline{X}(k) = \underline{X}(k-1) + \underline{U}(k) \tag{16.1}$$

where k = time interval

$\underline{U}(k)$ = white noise term that represents the change in the state of the system from one time interval to the next

$$E[\underline{U}(k)] = \underline{0}, \quad E[\underline{U}(k)\underline{U}'(k)] = S \tag{16.2}$$

The development of the Kalman filter will now proceed in two steps. In the first step, a linear measurement equation including a bias error will be assumed, and the Kalman filter will be derived for this model. In the second step, the actual nonlinear measurement equation will be considered, and the results of the first step will be extended to this model.

Step 1: Assume the following measurement equation,

$$\underline{Z}(k) = F(k)\underline{X}(k) + \underline{V}(k) + \underline{W} \tag{16.3}$$

where $E[\underline{V}(k)] = \underline{0}, \quad E[\underline{V}(k)\underline{V}'(k)] = R$ (16.4)

$$E[\underline{W}] = \underline{0}, \quad E[\underline{W}\underline{W}'] = Q \tag{16.5}$$

$\underline{V}(k)$ = white noise

$\underline{W}$ = bias error

We wish to find:

(1) An estimate, $\hat{\underline{X}}(k/k)$, such that

$$L = E[(\underline{X}(k) - \hat{\underline{X}}(k/k))'(\underline{X}(k) - \hat{\underline{X}}(k/k))] \tag{16.6}$$

is minimized.

(2) The covariance of $\hat{\underline{X}}(k/k)$,

$$P(k/k) = E[(\underline{X}(k) - \hat{\underline{X}}(k/k))(\underline{X}(k) - \hat{\underline{X}}(k/k))'] \tag{16.7}$$

(3) The correlation between $\hat{\underline{X}}(k/k)$ and $\underline{W}$,

$$D(k/k) = E[(\underline{X}(k) - \hat{\underline{X}}(k/k))\underline{W}'] \tag{16.8}$$

We are now interested in finding the best linear estimate that is of the following form,

$$\hat{\underline{X}}(k/k) = \hat{\underline{X}}(k/k-1) + A(k)(\underline{Z}(k) - \hat{\underline{Z}}(k/k-1)) \tag{16.9}$$

where $\hat{\underline{X}}(k/k-1)$ = The best estimate of $\underline{X}(k)$ given the data up to time k-1. In this case

$$\hat{\underline{X}}(k/k-1) = \hat{\underline{X}}(k-1/k-1) \tag{16.10}$$

$\underline{\hat{Z}}(k/k-1)$ = The best estimate of $\underline{Z}(k)$, computed from $\underline{\hat{X}}(k/k-1)$.
In this case

$$\underline{\hat{Z}}(k/k-1) = F(k)\underline{\hat{X}}(k/k-1) \tag{16.11}$$

$A(k)$ = Gain matrix to be determined so that the cost function, L, is minimized.

$$E[(\underline{X}(k) - \underline{\hat{X}}(k/k))(\underline{X}(k) - \underline{\hat{X}}(k/k))']$$

$$= E\{[(\underline{X}(k) - \underline{\hat{X}}(k/k-1) - A(k)(\underline{Z}(k) - \underline{\hat{Z}}(k/k-1))] \cdot$$

$$[(\underline{X}(k) - \underline{\hat{X}}(k/k-1)) - A(k)(\underline{Z}(k) - \underline{\hat{Z}}(k/k-1))]'\}$$

$$= E[(\underline{X}(k) - \underline{\hat{X}}(k/k-1))(\underline{\hat{X}}(k) - \underline{\hat{X}}(k/k-1))']$$

$$-E[A(k)(\underline{Z}(k) - \underline{\hat{Z}}(k/k-1))(\underline{X}(k) - \underline{\hat{X}}(k/k-1))']$$

$$-E[(\underline{X}(k) - \underline{\hat{X}}(k/k-1))(\underline{Z}(k) - \underline{\hat{Z}}(k/k-1))'A'(k)]$$

$$+E[A(k)(\underline{Z}(k) - \underline{\hat{Z}}(k/k-1))(\underline{Z}(k) - \underline{\hat{Z}}(k/k-1))'A'(k)] \tag{16.12}$$

Since $\underline{Z}(k) = F(k)\underline{X}(k) + \underline{V}(k) + \underline{W}$ and $\underline{\hat{Z}}(k/k-1) = F(k)\underline{\hat{X}}(k/k-1)$,

$$E[(\underline{Z}(k) - \underline{\hat{Z}}(k/k-1))(\underline{Z}(k) - \underline{\hat{Z}}(k/k-1))']$$

$$= E\{[F(k)(\underline{X}(k) - \underline{\hat{X}}(k/k-1)) + \underline{V}(k) + \underline{W}]$$

$$[F(k)(\underline{X}(k) - \underline{\hat{X}}(k/k-1)) + \underline{V}(k) + \underline{W}]'\}$$

$$= F(k)P(k/k-1)F'(k) + F(k)D(k/k-1) + D'(k/k-1)F'(k) + Q + R$$

$$\equiv H(k) \tag{16.13}$$

$$E[(\underline{X}(k) - \hat{\underline{X}}(k/k-1))(\underline{Z}(k) - \hat{\underline{Z}}(k/k-1))'A'(k)]$$

$$= E[(\underline{X}(k) - \hat{\underline{X}}(k/k-1))((\underline{X}(k) - \hat{\underline{X}}(k/k-1))'F'(k) + \underline{V}'(k) + \underline{W}')A'(k)]$$

$$= P(k/k-1)F'(k)A'(k) + D(k/k-1)A'(k) \tag{16.14}$$

$$E[A(k)(\underline{Z}(k) - \hat{\underline{Z}}(k/k-1))(\underline{X}(k) - \hat{\underline{X}}(k/k-1))']$$

$$= A(k)F(k)P(k/k-1) + A(k)D'(k/k-1) \tag{16.15}$$

Substituting Equations 16.13, 16.14, and 16.15 into Equation 16.12,

$$P(k/k) = P(k/k-1) - A(k)F(k)P(k/k-1) - A(k)D'(k/k-1)$$
$$- P(k/k-1)F'(k)A'(k) - D(k/k-1)A'(k) + A(k)H(k)A'(k) \tag{16.16}$$

The problem now is to determine A(k) such that the trace of Equation 16.16 is a minimum. Note that if A(k), P(k/k-1), F(k), D(k/k-1), and H(k) are scalars, we can take the derivative of Equation 16.16 with respect to A(k) and set the result = 0,

$$-F(k)P(k/k-1) - D'(k/k-1) - P(k/k-1)F'(k) - D(k/k-1) + 2A(k)H(k) = 0 \tag{16.17}$$

or

$$A(k) = (P(k/k-1)F'(k) + D(k/k-1))H(k)^{-1} \tag{16.18}$$

To prove that this is also the solution for the matrix case, we will set

$$A(k) = B + (P(k/k-1)F'(k) + D(k/k-1))H(k)^{-1} \tag{16.19}$$

and prove that the trace of Equation 16.16 is minimized for B = 0.

$$\mathrm{Tr}[P(k/k)] = \mathrm{Tr}\{P(k/k-1) - [B + (P(k/k-1)F'(k) + D(k/k-1))H(k)^{-1}]$$
$$[F(k)P(k/k-1) + D'(k/k-1)]$$
$$- [P(k/k-1)F'(k) + D(k/k-1)]$$
$$[H(k)^{-1}(F(k)P(k/k-1) + D'(k/k-1)) + B']$$
$$+ [B + (P(k/k-1)F'(k) + D(k/k-1))H(k)^{-1}](H(k))$$
$$[B' + H(k)^{-1}(F(k)P(k/k-1) + D'(k/k-1))]\} \quad (16.20)$$

After cancelling terms we have,

$$\mathrm{Tr}[P(k/k-1)] = \mathrm{Tr}[P(k/k-1) - (P(k/k-1)F'(k)$$
$$+ D(k/k-1))H(k)^{-1}(F(k)P(k/k-1)$$
$$+ D'(k/k-1)) + BH(k)B'] \quad (16.21)$$

$\mathrm{Tr}[P(k/k)] \geq 0$ and $\mathrm{Tr}[BH(k)B'] \geq 0$ therefore Equation 16.21 will be minimized for $\mathrm{Tr}[BH(k)B'] = 0$ or for $B = 0$.

Therefore,

$$P(k/k) = P(k/k-1) - (P(k/k-1)F'(k) + D(k/k-1))H(k)^{-1} \cdot$$
$$(F(k)P(k/k-1) + D'(k/k-1)) \quad (16.22)$$

The last term to be determined is

$$D(k/k) = E[(\underline{X}(k) - \hat{\underline{X}}(k/k))\underline{W}']$$
$$= E\{[(\underline{X}(k) - \hat{\underline{X}}(k/k-1)) - (P(k/k-1)F'(k)$$
$$+ D(k/k-1))H(k)^{-1}(F(k)(\underline{X}(k) - \hat{\underline{X}}(k/k-1)) + \underline{V}(k) + \underline{W})]\underline{W}'\}$$
$$(16.23)$$

Therefore,

$$D(k/k) = D(k/k-1) - (P(k/k-1)F'(k) + D(k/k-1))\,H(k)^{-1}\cdot$$
$$(F(k)D(k/k-1) + Q) \qquad (16.24)$$

The set of equations for the recursive estimator are summarized as follows:

$$\hat{\underline{X}}(k/k) = \hat{\underline{X}}(k/k-1) + (P(k/k-1)F'(k) + D(k/k-1))H(k)^{-1}\cdot$$
$$(\underline{Z}(k) - F(k)\hat{\underline{X}}(k/k-1)) \qquad (16.25)$$

$$P(k/k) = P(k/k-1) - (P(k/k-1)F'(k) + D(k/k-1))\,H(k)^{-1}\cdot$$
$$(F(k)P(k/k-1) + D'(k/k-1)) \qquad (16.26)$$

$$D(k/k) = D(k/k-1) - (P(k/k-1)F'(k) + D(k/k-1))H(k)^{-1}\cdot$$
$$(F(k)D(k/k-1) + Q) \qquad (16.27)$$

$$H(k) = F(k)P(k/k-1)F'(k) + F(k)D(k/k-1) + D'(k/k-1)F'(k) + Q + R \qquad (16.28)$$

Now,

$$\begin{aligned} P(k/k-1) &= E[(\hat{\underline{X}}(k/k-1) - \underline{X}(k))(\hat{\underline{X}}(k/k-1) - \underline{X}(k))'] \\ &= E[(\hat{\underline{X}}(k-1/k-1) - \underline{X}(k-1) - \underline{U}(k))(\hat{\underline{X}}(k-1/k-1) - \underline{X}(k-1) \\ &\quad - \underline{U}(k))'] \\ &= P(k-1/k-1) + S \qquad (16.29) \end{aligned}$$

$$D(k/k-1) = E[(\underline{X}(k) - \hat{\underline{X}}(k/k-1))\underline{W}']$$

$$= E[(\underline{X}(k-1) + \underline{U}(k) - \hat{\underline{X}}(k-1/k-1))\underline{W}']$$

$$= D(k-1/k-1) + E[\underline{U}(k)\underline{W}'] \tag{16.30}$$

Therefore if we assume $E[\underline{U}(k)\underline{W}'] = 0$, then,

$$D(k/k-1) = D(k-1/k-1) \tag{16.31}$$

Therefore in summary, Equation 16.25 through Equation 16.28 can be written,

$$\hat{\underline{X}}(k/k) = \hat{\underline{X}}(k-1/k-1) + [P(k-1/k-1)F'(k) + SF'(k) + D(k-1/k-1)]H(k)^{-1} \cdot [\underline{Z}(k) - F(k)\hat{\underline{X}}(k-1/k-1)] \tag{16.32}$$

$$P(k/k) = P(k-1/k-1) + S - [P(k-1/k-1)F'(k) + SF'(k) + D(k-1/k-1)]H(k)^{-1}[F(k)P(k-1/k-1) + F(k)S + D'(k-1/k-1)] \tag{16.33}$$

$$D(k/k) = D(k-1/k-1) - [P(k-1/k-1)F'(k) + SF'(k) + D(k-1/k-1)]H(k)^{-1}[F(k)D(k-1/k-1) + Q] \tag{16.34}$$

$$H(k) = F(k)[P(k-1/k-1) + S]F'(k) + F(k)D(k-1/k-1) + D'(k-1/k-1)F'(k) + Q + R \tag{16.35}$$

Step 2: Assume that we have a system described by the following nonlinear equations,[1]

[1] (16, Chapter 7).

$$\underline{X}(k+1) = \underline{g}(\underline{X}(k)) + \underline{U}(k) \tag{16.36}$$

$$\underline{Z}(k) = \underline{f}(\underline{X}(k)) + \underline{V}(k) + \underline{W} \tag{16.37}$$

If we know some $\overset{*}{\underline{X}}(k)$ sufficiently close to $\underline{X}(k)$, $\underline{f}(\underline{X}(k))$ and $\underline{F}(\underline{X}(k))$ can be represented by a Taylor series as follows,

$$\underline{g}(\underline{X}(k)) = \underline{g}(\overset{*}{\underline{X}}(k)) + G(k)[\underline{X}(k) - \overset{*}{\underline{X}}(k)] \tag{16.38}$$

$$\underline{f}(\underline{X}(k)) = \underline{f}(\overset{*}{\underline{X}}(k)) + F(k)[\underline{X}(k) - \overset{*}{\underline{X}}(k)] \tag{16.39}$$

where $G(k)$ = Jacobian of $\underline{g}(\overset{*}{\underline{X}}(k))$

$F(k)$ = Jacobian of $\underline{f}(\overset{*}{\underline{X}}(k))$

Let

$$\underline{X}_e(k) = \underline{X}(k) - \overset{*}{\underline{X}}(k) \tag{16.40}$$

$$\underline{X}_e(k+1) = \underline{g}(\overset{*}{\underline{X}}(k)) + G(k)\underline{X}_e(k) + \underline{U}(k) - \overset{*}{\underline{X}}(k+1) \tag{16.41}$$

$$\underline{Z}(k) = \underline{f}(\overset{*}{\underline{X}}(k)) + F(k)\underline{X}_e(k) + \underline{V}(k) + \underline{W} \tag{16.42}$$

Let $\underline{X}_g(k)$ be the deterministic solution of

$$\underline{X}_g(k+1) = G(k)\underline{X}_g(k) + \underline{g}(\overset{*}{\underline{X}}(k)) - \overset{*}{\underline{X}}(k+1) \tag{16.43}$$

Let

$$\underline{Y}(k) = \underline{X}_e(k) - \underline{X}_g(k) \tag{16.44}$$

$$\begin{aligned}\underline{Z}(k) - \underline{f}(\overset{*}{\underline{X}}(k)) - F(k)\underline{X}_g(k) &= \hat{F}(k)[\underline{X}_e(k) - \underline{X}_g(k)] + \underline{V}(k) + \underline{W} \\ &= F(k)\underline{Y}(k) + \underline{V}(k) + \underline{W}\end{aligned} \tag{16.45}$$

Now,

$$\underline{Y}(k+1) = \underline{X}_e(k+1) - \underline{X}_g(k+1) \tag{16.46}$$

Therefore we have for $\underline{Y}(k)$,

$$\underline{Y}(k+1) = G(k)\underline{Y}(k) + \underline{U}(k) \tag{16.47}$$

$$Z(k) - \underline{f}(\underline{X}^*(k)) - F(k)\underline{X}_g(k) = F(k)\underline{Y}(k) + \underline{V}(k) + \underline{W} \tag{16.48}$$

We can now solve for $\hat{\underline{Y}}(k/k)$,

$$\underline{X}(k) = \underline{Y}(k) + \underline{X}_g(k) + \underline{X}^*(k) \tag{16.49}$$

$$\hat{\underline{X}}(k/k) = \hat{\underline{Y}}(k/k) + \underline{X}_g(k) + \underline{X}^*(k) \tag{16.50}$$

Suppose that we have $\hat{\underline{X}}(k-1/k-1)$ and want to generate $\underline{X}^*(k)$.

Let,

$$\underline{X}^*(k) = \hat{\underline{X}}(k/k-1) = \underline{g}(\hat{\underline{X}}(k-1/k-1)) \tag{16.51}$$

Now,

$$\hat{\underline{X}}(k/k-1) = \hat{\underline{Y}}(k/k-1) + \underline{X}_g(k) + \hat{\underline{X}}(k/k-1) \tag{16.52}$$

which implies,

$$\hat{\underline{Y}}(k/k-1) = -\underline{X}_g(k) \tag{16.53}$$

Therefore,

$$\hat{\underline{X}}(k/k) = \hat{\underline{Y}}(k/k) - \hat{\underline{Y}}(k/k-1) + \hat{\underline{X}}(k/k-1) \tag{16.54}$$

or

$$\underline{\hat{X}}(k/k) - \underline{\hat{X}}(k/k-1) = \underline{\hat{Y}}(k/k) - \underline{\hat{Y}}(k/k-1) \tag{16.55}$$

Equation 16.48 is similar in form to Equation 16.3. Therefore from Equation 16.25,

$$\begin{aligned}\underline{\hat{Y}}(k/k) &= \underline{\hat{Y}}(k/k-1) + (P(k/k-1)F'(k) + D(k/k-1))H(k)^{-1}\cdot \\ &\quad (\underline{Z}(k) - \underline{f}(\underline{\hat{X}}(k/k-1)) + F(k)\underline{\hat{Y}}(k/k-1) - F(k)\underline{\hat{Y}}(k/k-1)) \\ &= \underline{\hat{Y}}(k/k-1) + (P(k/k-1)F'(k) + D(k/k-1))H(k)^{-1}\cdot \\ &\quad (\underline{Z}(k) - \underline{f}(\underline{\hat{X}}(k/k-1)))\end{aligned} \tag{16.56}$$

This completes step 1 and step 2. The actual system we are considering can be described by the following equations,

$$\underline{X}(k) = \underline{X}(k-1) + \underline{U}(k) \tag{16.57}$$

$$\underline{Z}(k) = \underline{f}(\underline{X}(k)) + \underline{V}(k) + \underline{W} \tag{16.58}$$

Equations 16.32 , 16.33, 16.34, and 16.35 can now be used to furnish the following recursive algorithm for the system described by Equations 16.57 and 16.58,

$$\begin{aligned}\underline{\hat{X}}(k/k) &= \underline{\hat{X}}(k-1/k-1) + [P(k-1/k-1)F'(k) + SF'(k) + D(k-1/k-1)]H(k)^{-1}\cdot \\ &\quad [\underline{Z}(k) - \underline{f}(\underline{\hat{X}}(k-1/k-1))]\end{aligned} \tag{16.59}$$

$$\begin{aligned}P(k/k) &= P(k-1/k-1) + S - [P(k-1/k-1)F'(k) + SF'(k) \\ &\quad + D(k-1/k-1)]H(k)^{-1}[F(k)P(k-1/k-1) + F(k)S + D'(k-1/k-1)]\end{aligned} \tag{16.60}$$

$$D(k/k) = D(k\text{-}1/k\text{-}1) - [P(k\text{-}1/k\text{-}1)F'(k) + SF'(k) + D(k\text{-}1/k\text{-}1)]H(k)^{-1}[F(k)D(k\text{-}1/k\text{-}1) + Q] \tag{16.61}$$

$$H(k) = F(k)[P(k\text{-}1/k\text{-}1) + S]F'(k) + F(k)D(k\text{-}1/k\text{-}1) + D'(k\text{-}1/k\text{-}1)F'(k) + Q + R \tag{16.62}$$

XVII. APPENDIX B: MEASUREMENT EQUATIONS FOR LINES OPEN AT ONE END

A logical approach for maintaining an up-to-date model for an on-line estimation program would be to monitor the status of the circuit breakers in the system and then revise the model to account for breakers that have changed position. Some of the literature[1] has suggested using the estimation program itself to perform this function via an anomaly detection scheme, but this seems to be a rather complicated approach in light of the fact that this information is almost always directly available at the system control center.

Thus lines can be opened or closed in the program to agree with the current topology of the physical system, but some question remains as to how these open lines should be accounted for. Referring to Figure 17.1, it can be seen that the open line can be represented by the following shunt admittance:

$$Y_E = Y_1 + \frac{Y_{12}\,Y_2}{Y_{12}+Y_2} = y_1 e^{-j\beta_1} + \frac{y_{12} y_2 e^{-j(\theta_{12}+\beta_2)}}{y_{12} e^{-j\theta_{12}} + y_2 e^{-j\beta_2}} = y_e e^{-j\beta_e} \tag{17.1}$$

[1]Reference 20.

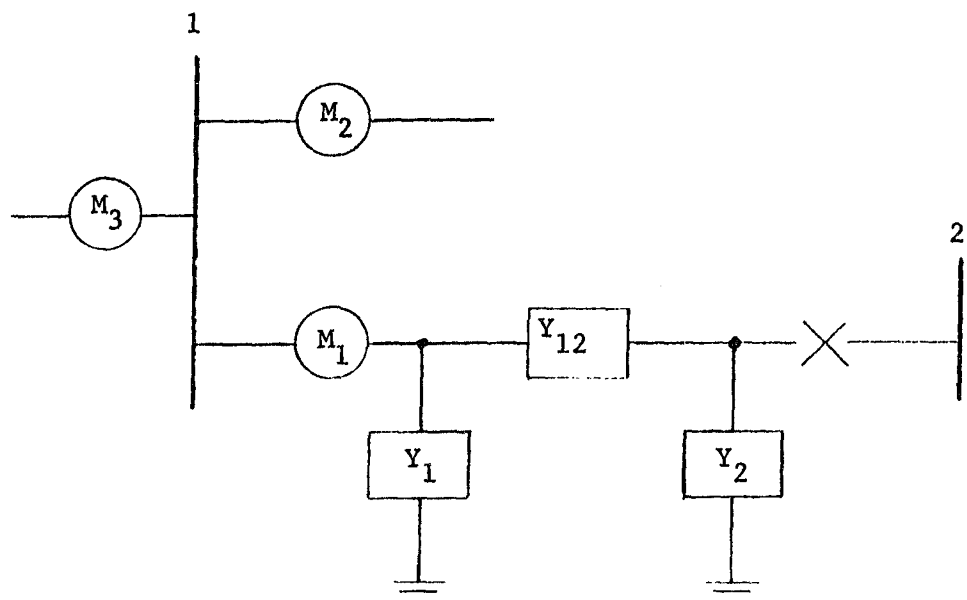


Figure 17.1. Line open at one end only

If M_1, M_2, and M_3 in Figure 17.1 are separate line flow measurements, the open line can easily be accounted for by eliminating M_1 from the set of measurement equations. Even with the line open however, M_1 can still provide some useful information about the voltage magnitude, e_1, as can be seen by the following equation,

$$g_1 + jh_1 = e_1 \angle\delta_1 (e_1 \angle\delta_1 \cdot y_e \angle -\beta_e)^*$$

$$= e_1^2 y_e \angle\beta_e = e_1^2 y_e \cos(\beta_e) + je_1^2 y_e \sin(\beta_e) \qquad (17.2)$$

Y_E must be calculated off-line and stored, so some effort will be involved in making use of this information. The question then is to determine if the information provided by Equation 17.2 justifies incorporating it into the on-line computer program.

If M_1, M_2, and M_3 are not measured separately but as the sum of the injected bus power, $M = M_1 + M_2 + M_3$, the alternatives are slightly different. The M_1 terms can easily be eliminated from the measurement equation by writing $M = M_2 + M_3$, but then the measured M is in error and the question is to determine if this error is significant. To be exact, the following equation should be used,

$$M = e_1^2 y_e \cos(\beta_e) + j e_1^2 y_e \sin(\beta_e) + M_2 + M_3 \tag{17.3}$$

XVIII. APPENDIX C: STORAGE LOCATION PROGRAM

The coding shown in this appendix is for the computer program that generates the codes necessary for storing the sparse matrices used in the STATE ESTIMATOR and SENSITIVITY ANALYSIS programs. For the IBM 360/65, the program requires 66 K bytes of main core memory when compiled in Fortran G.

```
C STORAGE LOCATION PROGRAM
C ALL VARIABLES ARE DEFINED IN COMMENTS OR WRITE STATEMENTS
C
      INTEGER*2 HEAD(70),TAIL(70),BUS(60),LINE(70),BUSV(60),SNCON(60),
     1 ENCON(60),CLINE(140),CBUS(140),LOOK1(175),LOOK2(175),LOOK(800),
     1 CODE1(120),CODE2(120),CODE(800), NCOM1(120),NCOM2(120),
     1 COM1(650),COM2(650),FCOM(1700),SCOM(1700),CSTATE(650),
     1 SBCOM(120),EBCOM(120),BCOM(650),LBCOM(650),COL(1400),
     1 SCOL(120),ECOL(120)
      COMMON/ONE/NMEAS,NSM,NSTATE,NELEM,LOOK1,LOOK2,LOOK,CODE1,
     1 CODE2,NCOM1,NCOM2,SBCOM,EBCOM,CODE,COM1,COM2,CSTATE,FCOM,
     1 SCOM,BCOM,LBCOM,COL,SCOL,ECOL
      READ(5,700) NBUS,NLINE
  700 FORMAT(I3,2X,I3)
      WRITE(6,701)  NBUS,NLINE
  701 FORMAT('0',2X,'NO. BUSES=',I3,2X,'NO. LINES=',I3)
      READ(5,44) NBUSM,NLINEM,NVOLTM
   44 FORMAT(    3(2X,I3))
      WRITE(6,45) NBUSM
   45 FORMAT('0',2X,'NO. OF REAL AND IMAG BUS PWR. MEAS.=',I3)
      WRITE(6,46) NLINEM
   46 FORMAT('0',2X,'NO. OF REAL AND IMAG LINE PWR. MEAS.=',I3)
      WRITE(6,47) NVOLTM
   47 FORMAT('0',2X,'NO. OF BUS VOLTAGE MEAS=',I3)
C
C BUS=LIST OF BUS INJECTION MEASUREMENTS
C
      READ(5,1) (BUS(K),K=1,NBUSM)
    1 FORMAT(T5,I3)
      WRITE(6,2)
    2 FORMAT('0',T5,'BUS',T42,'BUS')
      WRITE(6,60) (BUS(K),K=1,NBUSM)
   60 FORMAT('0',T5,I3,T42,I3)
      M=2*NBUSM
      N=M+NLINEM
C
C LINE=LIST OF LINE FLOW MEASUREMENTS
```

```
C HEAD=LIST OF BUSES WHERE LINE FLOW MEAS. IS MADE
C TAIL=LIST OF BUSES AT END OF LINE OPPOSITE TO HEAD
C
      READ(5,4) (LINE(K),HEAD(K),TAIL(K),K=1,NLINEM)
    4 FORMAT(2X,I3,T8,I3,T13,I3)
      WRITE(6,5)
    5 FORMAT('0',2X,'LINE',T8,'HEAD',T13,'TAIL')
      WRITE(6,6) (LINE(K),HEAD(K),TAIL(K),K=1,NLINEM)
    6 FORMAT('0',2X,I3,T8,I3,T13,I3)
      N=2*(NBUSM+NLINEM)
C
C BUSV=LIST OF BUSES WHERE VOLTAGE IS MEASURED.
C
      READ(5,7) (BUSV(K),K=1,NVOLTM)
    7 FORMAT(T5,I3)
      WRITE(6,8)
    8 FORMAT('0',7(2X,'BUS'))
      WRITE(6,9) (BUSV(K),K=1,NVOLTM)
    9 FORMAT('0',7(2X,I3))
C
C CLINE=LIST OF LINES CONNECTED TO EACH BUS
C CBUS=LIST OF BUSES AT OPPOSITE END OF EACH CLINE
C SNCON=FIRST ELEMENT  IN CLINE LIST FOR EACH BUS
C ENCON=LAST ELEMENT IN CLINE LIST FOR EACH BUS
C
      READ(5,875) (SNCON(K),ENCON(K),K=1,NBUS)
  875 FORMAT(T5,I3,T10,I3)
      DO 928 K=1,NBUS
      M=SNCON(K)
      N=ENCON(K)
      READ(5,929) (CBUS(L),L=M,N)
  929 FORMAT(T15,10(I3,2X))
      READ(5,876) (CLINE(J),J=M,N)
  876 FORMAT(T15,10(I3,2X))
      WRITE(6,931)
  931 FORMAT('0',1X,'BUS',T8,'START',T16,'END',T26,' BUS CONNECT',T66,
     1 'LINE CONNECT')
```

```
      WRITE(6,930) K,SNCON(K),ENCON(K),CBUS(M),CLINE(M)
  930 FORMAT('0',1X,I3,T8,I3,T16,I3,T26,I3,T66,I3)
      IF(M.EQ.N) GO TO 928
      JK=M+1
      WRITE(6,296) (CBUS(L),CLINE(L),L=JK,N)
  296 FORMAT(' ',T26,I3,T66,I3)
  928 CONTINUE
C
C NSTATE=NO. OF STATES
C NMEAS=NO. OF MEASUREMENTS
C
      NSTATE=2*NBUS-1
      NSM=NSTATE-1
      NMEAS=2*(NBUSM+NLINEM)+NVOLTM
C
C KHH=INDEX FOR TYPE OF MEASUREMENT
C M=INDEX FOR LISTING STATES
C LOOK=LIST OF STATES AS THEY APPEAR IN THE EQUATION FOR EACH MEASUREMENT
C LOOK1=LOCATION OF FIRST ELEMENT IN LOOK LIST FOR EACH MEAS.
C LOOK2=LOCATION OF LAST ELEMENT IN LOOK LIST FOR EACH MEAS.
C
      KHH=0
      M=0
      DO 500 II=1,2
      DO 501 K=1,NBUSM
      KBUS=BUS(K)
      KK=SNCON(KBUS)
      LL=ENCON(KBUS)
      LOOK1(K+KHH)=M+1
      M=M+1
      LOOK(M)=BUS(K)
      IF(KBUS.GE.NBUS) GO TO 503
      M=M+1
      LOOK(M)=KBUS+NBUS
  503 CONTINUE
      DO 502 L=KK,LL
      LCBUS=CBUS(L)
```

```
      M=M+1
      LOOK(M)=LCBUS
      IF(LCBUS.GE.NBUS) GO TO 502
      M=M+1
      LOOK(M)=LCBUS+NBUS
  502 CONTINUE
      LOOK2(K+KHH)=M
  501 CONTINUE
      KHH=NBUSM
  500 CONTINUE
      WRITE(6,600)
  600 FORMAT('0','PASSED 500 OK')
      KHH=2*NBUSM
      DO 505 II=1,2
      DO 506 K=1,NLINEM
      KTAIL=TAIL(K)
      KHEAD=HEAD(K)
      M=M+1
      LOOK1(K+KHH)=M
      LOOK(M)=KHEAD
      IF(KHEAD.GE.NBUS) GO TO 508
      M=M+1
      LOOK(M)=KHEAD+NBUS
  508 M=M+1
      LOOK(M)=KTAIL
      IF(KTAIL.GE.NBUS) GO TO 509
      M=M+1
      LOOK(M)=KTAIL+NBUS
  509 LOOK2(K+KHH)=M
  506 CONTINUE
      KHH=KHH+NLINEM
  505 CONTINUE
      WRITE(6,601)
  601 FORMAT('0','PASSED 505 OK')
      KHH=2*(NBUSM+NLINEM)
      DO 535 K=1,NVOLTM
      M=M+1
```

```
      LCOK1(K+KHH)=M
      LOCK2(K+KHH)=M
      LCCK(M)=BUSV(K)
  535 CCNTINUE
C
C M=INDEX FOR LISTING MEASUREMENTS
C KAD=0, INDEX FOR LISTINGS PERTAINING TO EACH BUS VOLTAGE
C KAD=NBUS, INDEX FOR LISTINGS PERTAINING TO THE PHASE ANGLE AT EACH BUS
C CODE=LIST CF MEASUREMENTS AS THEY INCLUDE EACH OF THE STATES
C CODE1=LOCATICN OF FIRST ELEMENT IN CODE LIST FOR EACH STATE
C CODE2=LCCATION OF LAST ELEMENT IN CODE LIST FOR EACH STATE
C KEY=KEY WORC THAT PREVENTS INCLUDING WRONG ELEMENTS IN CERTAIN LISTS
C
      M=0
      NNBUSM=2*NBUSM
      KJC=NBUS
      KAD=0
      KBULIN=2*(NBUSM+NLINEM)
C LOOP FOR STATES 1 THRU NBUS AND THEN FOR NBUS+1 THRU 2*NBUS-1
      DO 512 IM=1,2
      DO 511 K=1,KJC
      CODE1(K+KAD)=M+1
      KK=SNCON(K)
      LL=ENCON(K)
      KHH=0
C
C  LCOP FOR REAL AND THEN REACTIVE BUS POWER
C
      DO 513 JK=1,2
      KEY=C
      DO 514 L=KK,LL
      LCBUS=CBLS(L)
      IF(LCBUS.LT.K) GO TO 515
      IF(KEY.GT.0) GC TO 515
      KEY=1
      CO 516 I=1,NBUSM
      JJ=I
```

```
      IBUS=BUS(I)
      IF(IBUS.EQ.K) GO TO 517
      IF(IBUS.GT.K) GO TO 515
  516 CONTINUE
      GO TO 515
  517 M=M+1
      CODE(M)=JJ+KHH
  515 CONTINUE
      DO 518 I=1,NBUSM
      JJ=I
      IBUS=BUS(I)
      IF(LCBUS.EQ.IBUS) GO TO 519
      IF(LCBUS.LE.IBUS) GO TO 514
  518 CONTINUE
      GO TO 514
  519 M=M+1
      CODE(M)=JJ+KHH
  514 CONTINUE
      IF(KEY.GT.0 ) GO TO  575
      DO 576 I=1,NBUSM
      JJ=I
      IBUS=BUS(I)
      IF(IBUS.EQ.K) GO TO 577
      IF(IBUS.GT.K) GO TO 575
  576 CONTINUE
      GO TO 575
  577 M=M+1
      CODE(M)=JJ+KHH
  575 KHH=NBUSM
  513 CONTINUE
      WRITE(6,602)
  602 FORMAT('0','PASSED 513 OK')
      KKH=0
C
C LOOP FOR REAL AND THEN REACTIVE LINE POWER
C
      DO 520 IJ=1,2
```

```
C
C KEEP=INDEX FOR GENERATING LINE FLOW CODE ELEMENTS
C
      KEEP=NNBUSM+KKH
      DO 521 L=1,NLINEM
      JJ=L
      IF(HEAD(L).EQ.K) GO TO 522
      IF(TAIL(L).NE.K) GO TO 521
  522 M=M+1
      CODE(M)=JJ+KEEP
  521 CONTINUE
      KKH=NLINEM
  520 CONTINUE
      IF((K+KAD).GT.BUSV(NVOLTM)) GO TO 525
      DO 523 L=1,NVOLTM
      JJ=L
      LBUSV=BUSV(L)
      IF(K.EQ.LBUSV) GO TO 524
      IF(K.LT.LBUSV) GO TO 525
  523 CONTINUE
      GO TO 525
  524 M=M+1
      CODE(M)=KBULIN+JJ
  525 CONTINUE
      CODE2(K+KAD)=M
  511 CONTINUE
      KJC=NBUS-1
      KAD=NBUS
  512 CONTINUE
      WRITE(6,603)
  603 FORMAT('0','PASSED 512 OK')
C
C  CALC OF FCOM,SCOM,NCOM,AND COM AND CSTATE
C
C M=INDEX FOR COUNTING ELEMENTS IN CERTAIN LISTS
C CSTATE: FOR EACH STATE, CSTATE LISTS THE HIGHER NUMBERED STATES HAVING CODE
C ELEMENTS IN COMMON WITH THAT STATE
```

```
C FCOM: FOR EACH STATE, FCOM LISTS THE LOCATION OF ELEMENTS IN THE CODE LIST
C THAT ARE CCMMON TO THE CODE LIST ELEMENTS LISTED UNDER HIGHER NUMBERED STATES
C SCOM: SCCM LISTS THE STORAGE LOCATION OF THOSE ELEMENTS IN CODE WHOSE
C LOCATIONS WERE LISTED IN FCOM
C CCM1=FIRST ELEMENT IN FCOM AND SCOM LISTS FOR EACH CSTATE (LOCATION OF)
C COM2=LAST ELEMENT IN FCOM AND SCOM LISTS FCR EACH CSTATE (LOCATION OF)
C NCOM1=FIRST ELEMENT IN CSTATE LIST FOR EACH STATE (LOCATION OF)
C NCOM2=LAST ELEMENT IN CSTATE LIST FOR EACH STATE (LOCATION OF)
C KCOM=INDEX FCR GENERATING COM1 AND COM2
C KNCCM=INDEX FOR GENERATING NCOM1 AND NCOM2
C BCOM:FOR EACH STATE, BCOM LISTS THE LOWER NUMBERED STATES HAVING CODE
C ELEMENTS IN COMMON WITH THE CODE ELEMENTS FOR THAT STATE
C LBCCM LISTS THE LOCATION OF THE BCOM ELEMENTS IN THE CSTATE LIST
C SBCCM LISTS THE LOCATICN OF THE FIRST ELEMENT IN THE BCCM AND LBCOM LISTS FOR
C EACH STATE
C EBCCM LISTS THE LOCATION OF THE LAST ELEMENT IN THE BCOM AND LBCOM LISTS FOR
C EACH STATE
C KEY=KEY WORD THAT PREVENTS INCLUDING WRONG ELEMENTS IN CERTAIN LISTS
C COL LISTS THE COLUMNS FOR THE ELEMENTS OF EACH ROW OF THE COEFFICIENT MATRIX
C THAT WILL BE GENERATED IN THE STATE ESTIMATOR PROGRAM
C
      M=0
      KCOM=0
      KNCOM=0
      DO 526 K=1,NSM
      NCCM1(K)=KNCOM+1
      K1=K+1
      KCODE1=CODE1(K)
      KCODE2=CODE2(K)
      DO 527 L=K1,NSTATE
      KEY=0
      LCODE1=CODE1(L)
      LCODE2=CODE2(L)
      COM1(KNCOM+1)=KCCM+1
      DO 528 KK=KCODE1,KCODE2
      DO 529 LL=LCODE1,LCODE2
      JJ=LL
```

```
      IF(CODE(KK).EQ.CODE(LL)) GO TO 530
      IF(CODE(KK).LT.CODE(LL)) GO TO 528
  529 CONTINUE
      GO TO 528
  530 M=M+1
      FCOM(M)=KK
      SCOM(M)=JJ
      KCOM=KCOM+1
      IF(KEY.GT.0) GO TO 528
      KEY=1
      KNCOM=KNCOM+1
  528 CONTINUE
      IF(KNCOM.LT.1) GO TO 527
      COM2(KNCOM)=KCOM
      IF(KEY.GT.0) CSTATE(KNCOM)=L
  527 CONTINUE
      NCOM2(K)=KNCOM
      IF(NCOM1(K).GT.NCOM2(K)) NCOM1(K)=0
  526 CONTINUE
      WRITE(6,604)
  604 FORMAT('0','PASSED 526 OK')
      SBCOM(1)=0
      EBCOM(1)=0
      M=0
      DO 86 K=2,NSTATE
      SBCOM(K)=M+1
      K1=K-1
      DO 87 L=1,K1
      KK=NCOM1(L)
      LL=NCOM2(L)
      IF(KK.LT.1) GO TO 87
      DO 88 J=KK,LL
      JJ=J
      IF(CSTATE(J).EQ.K) GO TO 89
      IF(CSTATE(J).GT.K) GO TO 87
   88 CONTINUE
      GO TO 87
```

```
   89 M=M+1
      BCOM(M)=L
      LBCOM(M)=JJ
   87 CONTINUE
      EBCOM(K)=M
      IF(SBCOM(K).GT.EBCOM(K)) SBCOM(K)=0
   86 CONTINUE
C
C SCOL=LOCATION OF THE FIRST ELEMENT IN COL FOR EACH STATE
C ECOL=LOCATION OF THE LAST ELEMENT IN COL FOR EACH STATE
C
      MMM=NCOM1(1)
       M =NCOM2(1)
      SCOL(1)=1
      ECOL(1)=M
      IF(MMM.LT.1) GO TO 899
      DO 550 K=MMM,M
      COL(K)=CSTATE(K)
  550 CONTINUE
  899 CONTINUE
      DO 551 K=2,NSM
      N=0
      KSBCOM=SBCOM(K)
      KEBCOM=EBCOM(K)
      IF(KSBCOM.LT.1) GO TO 553
      DO 552 L=KSBCOM,KEBCOM
      M=M+1
      N=N+1
      COL(M)=BCOM(L)
  552 CONTINUE
  553 CONTINUE
      KNCOM1=NCOM1(K)
      KNCOM2=NCOM2(K)
      IF(KNCOM1.LT.1) GO TO 999
      DO 554 L=KNCOM1,KNCOM2
      M=M+1
      N=N+1
```

```
      CCL(M)=CSTATE(L)
  554 CONTINUE
  999 SCOL(K)=ECOL(K-1)+1
      ECOL(K)=ECOL(K-1)+N
  551 CONTINUE
      N=0
      NSBCOM=SBCOM(NSTATE)
      NEBCOM=EBCOM(NSTATE)
      DO 556 K=NSBCOM,NEBCOM
      N=N+1
      M=M+1
      CCL(M)=BCOM(K)
  556 CONTINUE
      SCOL(NSTATE)=ECOL(NSM)+1
      ECOL(NSTATE)=ECOL(NSM)+N
C
C NELEM = NO. OF OFF-DIAGONAL ELEMENTS IN COEFFICIENT MATRIX THAT WILL BE
C GENERATED IN THE STATE ESTIMATOR PROGRAM
C
      NELEM=M
      DO 814 K=1,NMEAS
      MA=LOOK1(K)
      MB=LOOK2(K)
      WRITE(6,816)
  816 FORMAT('0',1X,'MEAS',T8,'LOOK1',T20,'LOOK2',T30,'LOOK')
      WRITE(6,817) K,MA,MB,LOOK(MA)
  817 FORMAT('0',2X,I3,T9,I3,T21,I3,T31,I3)
      IF(MA.EQ.MB) GO TO 814
      JK=MA+1
      WRITE(6,818) (LOOK(L),L=JK,MB)
  818 FORMAT(' ',T31,I3)
  814 CONTINUE
      DO 819 K=1,NSM
      MA=CODE1(K)
      MB=CODE2(K)
      WRITE(6,825)
  825 FORMAT('0',1X,'STATE',T8,'CODE1',T14,'CODE2',T20,'CODE')
```

```
      WRITE(6,826) K,MA,MB,CODE(MA)
  826 FORMAT('0',2X,I3,T9,I3,T15,I3,T21,I3)
      IF(MA.EQ.MB) GO TO 819
      JK=MA+1
      WRITE(6,827) (CODE(L),L=JK,MB)
  827 FORMAT(' ',T21,I3)
  819 CONTINUE
      JA=CODE1(NSTATE)
      JB=CODE2(NSTATE)
      WRITE(6,840) K,JA,JB,CODE(JA)
  840 FORMAT('0',2X,I3,T9,I3,T15,I3,T21,I3)
      IF(JA.EQ.JB) GO TO 841
      JK=JA+1
      WRITE(6,842) (CODE(L),L=JK,JB)
  842 FORMAT(' ',T21,I3)
  841 CONTINUE
      DO 828 K=1,NSM
      MA=NCOM1(K)
      MB=NCOM2(K)
      WRITE(6,829)
  829 FORMAT('0',1X,'STATE',T8,'NCOM1',T14,'NCOM2',T20,'COM1',T25,
     1 'COM2',T30,'FCOM',T35,'SCOM',T42,'CSTATE')
      IF(MA.LT.1) GO TO 828
      WRITE(6,830) K,MA,MB,COM1(MA),COM2(MA),FCOM(COM1(MA)),
     1 SCOM(COM1(MA)),CSTATE(MA)
  830 FORMAT('0',2X,I3,T9,I3,T15,I3,T21,I4,T26,I4,T31,I3,T36,I3,T43,I3)
      NA=COM1(MA)
      NB=COM2(MA)
      IF(NA.EQ.NB) GO TO 832
      JK=NA+1
      WRITE(6,831) (FCOM(I),SCOM(I),I=JK,NB)
  831 FORMAT('0',T30,I3,T36,I3)
  832 IF(MA.EQ.MB) GO TO 828
      JK=MA+1
      DO 833 NN=JK,MB
      IA=COM1(NN)
      IB=COM2(NN)
```

```
      WRITE(6,834) IA,IB,FCCM(IA),SCOM(IA),CSTATE(NN)
  834 FCRMAT('0',T21,I4,T26,I4,T31,I3,T36,I3,T42,I3)
      IF(IA.EQ.IB) GC TO 833
      JT=IA+1
      WRITE(6,835) (FCOM(I),SCOM(I),I=JT,IB)
  835 FORMAT('0',T30,I3,T36,I3)
  833 CONTINUE
  828 CCNTINUE
      DO 20 K=2,NSTATE
      M=SBCCM(K)
      N=EBCOM(K)
      WRITE(6,21)
   21 FCRMAT('0',1X,'STATE',T8,'SBCOM',T16,'EBCOM',T26,'BCOM',T36,
     1 'LBCOM')
      IF(SBCOM(K).LT.1) GO TO 20
      WRITE(6,22) K,SBCCM(K),EBCOM(K),BCOM(M),LBCOM(M)
   22 FCRMAT('0',1X,I3,T8,I3,T16,I3,T26,I4,T36,I4)
      IF(M.EQ.N) GC TO 20
      JK=M+1
      WRITE(6,23) (BCCM(L),LBCOM(L),L=JK,N)
   23 FORMAT(' ',T26,I4,T36,I4)
   20 CCNTINUE
      WRITE(6,557)
  557 FORMAT('0',1X,'COL')
      WRITE(6,558) (COL(K),K=1,NELEM)
  558 FORMAT('0',20(1X,I3))
      WRITE(6,584) (SCCL(K),ECOL(K),K=1,NSTATE)
  584 FCRMAT('0',15(1X,I4))
      CALL PUNOUT
      STOP
      END


      SUBROUTINE PUNCUT
C
C THIS SUBROUTINE PUNCHES THE OUTPUT CODES
```

```
C
      INTEGER*2 HEAD(70),TAIL(70),BUS(60),LINE(70),BUSV(60),SNCON(60),
     1 ENCON(60),CLINE(140),CBUS(140),LOOK1(175),LOOK2(175),LOOK(800),
     1 CODE1(120),CODE2(120),CODE(800), NCOM1(120),NCOM2(120),
     1 COM1(650),COM2(650),FCOM(1700),SCOM(1700),CSTATE(650),
     1 SBCOM(120),EBCOM(120),BCOM(650),LBCOM(650),COL(1400),
     1 SCOL(120),ECOL(120)
      COMMON/ONE/NMEAS,NSM,NSTATE,NELEM,LOOK1,LOOK2,LOOK,CODE1,
     1 CODE2,NCOM1,NCOM2,SBCOM,EBCOM,CODE,COM1,COM2,CSTATE,FCOM,
     1 SCOM,BCOM,LBCOM,COL,SCOL,ECOL
      WRITE(7,100) (LOOK1(K),LOOK2(K),K=1,NMEAS)
  100 FORMAT(10(I3,2X,I3))
      DO 130 K=1,NMEAS
      MA=LOOK1(K)
      MB=LOOK2(K)
      WRITE(7,101) (LOOK(L),L=MA,MB)
  101 FORMAT(20(I3,1X))
  130 CONTINUE
      WRITE(7,102) (CODE1(K),CODE2(K),NCOM1(K),NCOM2(K),SBCOM(K),
     1 EBCOM(K),K=1,NSM)
  102 FORMAT(I3,T6,I3,T11,I3,T16,I3,T21,I3,T26,I3)
      WRITE(7,103) CODE1(NSTATE),CODE2(NSTATE),SBCOM(NSTATE),
     1 EBCOM(NSTATE)
  103 FORMAT(I3,T6,I3,T21,I3,T26,I3)
      DO 104 K=1,NSM
      MA=CODE1(K)
      MB=CODE2(K)
      WRITE(7,105) (CODE(L),L=MA,MB)
  105 FORMAT(20(I3,1X))
      NA=NCOM1(K)
      NB=NCOM2(K)
      IF(NA.LT.1) GO TO 104
      WRITE(7,106) (COM1(L),COM2(L),CSTATE(L),L=NA,NB)
  106 FORMAT(16(I4,1X))
      DO 107 J=NA,NB
      IA=COM1(J)
      IB=COM2(J)
```

```
      WRITE(7,108) (FCCM(L),SCOM(L),L=IA,IB)
  108 FCRMAT(20(I3,1X))
  107 CCNTINUE
  104 CCNTINUE
      DC 110 K=2,NSTATE
      KK=SBCCM(K)
      LL=EBCCM(K)
      IF(KK.LT.1) GO TO 110
      WRITE(7,111) (BCCM(L),LBCOM(L),L=KK,LL)
  111 FCRMAT(16(I4,1X))
  110 CCNTINUE
      JA=CCDE1(NSTATE)
      JB=CCDE2(NSTATE)
      WRITE(7,109) (CODE(L),L=JA,JB)
  109 FORMAT(20(I3,1X))
      WRITE(7,580) NELEM
  580 FCRMAT(I4)
      WRITE(7,582) (COL(K),K=1,NELEM)
  582 FCRMAT(20(I3,1X))
      WRITE(7,583) (SCOL(K),ECOL(K),K=1,NSTATE)
  583 FCRMAT(15(I4,1X))
      RETURN
      END
```

XIX. APPENDIX D: STATE ESTIMATOR PROGRAM

The coding shown in this appendix is for the computer program that calculates the state estimates from simulated system measurements. For the IBM 360/65, the program required 120 K bytes of main core memory when compiled in Fortran H.

```
C STATE ESTIMATOR PROGRAM
C
       REAL INCCV(175),YMAG(70),YANG(70),SUMRY(58),SUMIY(58),RY(70),
      1 IY(70),Z(175),        VOLT(58),         ANGLE(58),F(750),
      1 TEMPR(750)
       REAL   ELEM(1200),GAIN(5500),COST(2)
       REAL*8                DF,DF1,DF2,DF3,AC,CV,DD,EE,SRH,
      1 TGAIN,C(175),SUB,RHS(120),EWORK(120),FWORK(120),AEWORK,GWORK,
      1TDIAG,DIAG(120),AFWORK,DIAGK,RHSK,RHSL,ADIAG
       INTEGER*2 HEADY(70),COL(1200),SGAIN(120),EGAIN(120),SCOL(120),
      1 TAILY(70),HEAD(60),TAIL(60),BUS(60),LINE(60),BUSV(60),SNCON(60),
      1 ENCCN(60),CLINE(140),CBUS(140),LOOK1(175),LOOK2(175),LOOK(750),
      1 CODE1(120),CODE2(120),CODE(750),NCCM1(120),NCOM2(120),COM1(600),
      1 CCM2(600),FCOM(1600),SCOM(1600),LIL(120),CSTATE(600),SBCOM(120),
      1 EBCOM(120),BCOM(600),LBCOM(600),ECOL(120),CEWORK(120),CFWORK(120)
       INTEGER*2 COLG(5500)
       CCMMCN/CNE/C,NBUSM,VOLT,BUS,SUMRY,SUMIY,NLINE,YMAG,YANG,ANGLE,
      1 NLINEM,RY,IY,NVCLTM,HEADY,TAILY,HEAD,TAIL,LINE,BUSV,SNCON,ENCON,
      1 CLINE,CBUS
       CCMMCN/TWO/DF,DF1,DF2,DF3,AC,CV,DD,EE,SRH,TGAIN,SUB,RHS,EWORK,
      1 FWORK,AEWORK,GWORK,TDIAG,DIAG,AFWORK,DIAGK,RHSK,RHSL,ADIAG,
      1 INCOV,Z,F,TEMPR,ELEM,GAIN,NBUS,NMEAS,NSTATE,NSM,COL,SGAIN,EGAIN,
      1 SCCL,LCCK1,LOCK2,LOOK,CODE1,CODE2,CODE,NCOM1,NCOM2,COM1,COM2,
      1 FCOM,SCCM,LIL,CSTATE,SBCOM,EBCOM,BCOM,LBCOM,ECOL,CEWORK,CFWORK,
      1 COLG
C
C TIME, STARTM, STOPTM  ARE TIMING PROGRAMS STORED AT THE ISU IBM/360/65
C FACILITY. THESE PROGRAMS MAY BE OMITTED IF DESIRED.
C
       CALL TIME(S)
       T=0.0
       CALL STARTM(T)
C
C NBUS=NO. OF BUSES
C NLINE=NO. OF LINES
C
       READ(5,700) NBUS,NLINE
```

```
  700 FORMAT(I3,2X,I3)
      WRITE(6,701)  NBUS,NLINE
  701 FORMAT('0',2X,'NO. BUSES=',I3,2X,'NO. LINES=',I3)
C
C HEADY=BUS NO. AT THE END OF A LINE DEFINED TO BE THE HEAD FOR ADMITTANCE
C CALCULATIONS
C TAILY=BUS NO. AT THE END OF A LINE DEFINED TO BE THE TAIL FOR ADMITTANCE
C CALCULATIONS
C YMAG=MAGNITUDE OF THE SERIES ADMITTANCE OF EACH LINE
C YANG=PHASE ANGLE OF THE SERIES ADMITTANCE OF EACH LINE
C
      READ(5,702) (HEADY(K),TAILY(K),YMAG(K),YANG(K),K=1,NLINE)
  702 FORMAT(6X,I4,1X,I4,5X,E14.7,5X,E14.7)
      WRITE(6,703)
  703 FORMAT('0',1X,'LINE',1X,'HEAD',1X,'TAIL',7X,'YMAG',15X,'YANG')
      WRITE(6,704) (K,HEADY(K),TAILY(K),YMAG(K),YANG(K),K=1,NLINE)
  704 FORMAT('0',1X,I4,1X,I4,1X,I4,5X,E14.7,5X,E14.7)
C
C SUMRY=SUM OF THE REAL ADMITTANCES OF ALL LINES CONNECTED TO EACH BUS
C SUMIY=SUM OF THE REACTIVE ADMITTANCES OF ALL LINES CONNECTED TO EACH BUS
C
      READ(5,705) (SUMRY(K),SUMIY(K),K=1,NBUS)
  705 FORMAT(10X,E14.7,5X,E14.7)
      WRITE(6,40)
   40 FORMAT('0',3(4X,'BUS',2X,'REAL SUM',9X,'IMAG SUM'))
      WRITE(6,41) (K,SUMRY(K),SUMIY(K),K=1,NBUS)
   41 FORMAT('0',3(2X,I3,2X,E14.7,2X,E14.7))
C
C NBUSM=NO. OF REAL (OR REACTIVE) BUS INJECTION MEASUREMENTS
C NLINEM=NO. OF REAL (OR REACTIVE) LINE FLOW MEASUREMENTS
C NVOLTM=NO. OF BUS VOLTAGE MEASUREMENTS
C
      READ(5,44) NBUSM,NLINEM,NVOLTM
   44 FORMAT(    3(2X,I3))
      WRITE(6,45) NBUSM
   45 FORMAT('0',2X,'NO. OF REAL AND IMAG BUS PWR. MEAS.=',I3)
      WRITE(6,46) NLINEM
```

```
   46 FORMAT('0',2X,'NO. OF REAL AND IMAG LINE PWR. MEAS.=',I3)
      WRITE(6,47) NVCLTM
   47 FORMAT('0',2X,'NC. OF BUS VOLTAGE MEAS=',I3)
C
C INSET: CCUNTS THE SETS OF MEASUREMENTS THAT HAVE BEEN READ
C
      INSET=0
  900 CONTINUE
C
C BUS=BUS NC. OF EACH REAL (OR REACTIVE) BUS INJECTICN MEASUREMENT
C INCOV(K)=INVERSE VARIANCE OF EACH REAL BUS INJ. MEAS. ERROR
C INCOV(NBUSM+K)=INVERSE VARIANCE OF EACH REACTIVE BUS INJ. ERROR
C
      READ(5,1) (BUS(K),INCOV(K),INCOV(NBUSM+K),K=1,NBUSM)
    1 FORMAT(T5,I3,T16,E10.3,T28,E10.3)
      WRITE(6,2)
    2 FORMAT('0',T5,'BUS',T16,'RE INCOV',T28,'IM INCOV',T42,'BUS',T53,
     1 'RE INCOV',T65,'IM INCOV')
      WRITE(6,60) (BUS(K),INCOV(K),INCOV(NBUSM+K),K=1,NBUSM)
   60 FORMAT('0',T5,I3,T16,E10.3,T28,E10.3,T42,I3,T53,E10.3,T65,E10.3)
      M=2*NBUSM
      N=M+NLINEM
C
C LINE = LINE NO. OF EACH REAL (OR REACTIVE) LINE FLOW MEASUREMENT
C HEAD = END CF LINE WHERE LINE FLOW MEASUREMENT IS MADE
C TAIL = END CF LINE OPPOSITE FROM HEAD
C INCCV(M+K) = INVERSE VARIANCE OF EACH REAL LINE FLOW MEAS.
C INCOV(N+K) = INVERSE VARIANCE OF EACH REACTIVE LINE FLOW MEAS.
C
      READ(5,4) (LINE(K),HEAD(K),TAIL(K),INCOV(M+K),INCOV(N+K),K=1,
     1 NLINEM)
    4 FORMAT(2X,I3,T8,I3,T13,I3,T18,E10.3,T30,E10.3)
C
C RY = REAL ADMITTANCE OF EACH LINE THAT HAS A LINE FLOW MEASUREMENT
C IY = SUM CF THE SERIES AND SHUNT REACTIVE ADMITTANCE OF EACH LINE THAT HAS A
C LINE FLOW MEASUREMENT
C
```

```
      READ(5,706) (RY(K),IY(K),K=1,NLINEM)
  706 FORMAT(T42,E14.7,5X,E14.7)
      WRITE(6,5)
    5 FORMAT('0',2X,'LINE',T8,'HEAD',T13,'TAIL',T18,'RE INCOV',T30,
     1 'IM INCOV',T42,'RE Y',T60,'IM Y')
      WRITE(6,6) (LINE(K),HEAD(K),TAIL(K),INCOV(M+K),INCOV(N+K),RY(K),
     1 IY(K),K=1,NLINEM)
    6 FORMAT('0',2X,I3,T8,I3,T13,I3,T18,E10.3,T30,E10.3,T42,E14.7,T60,
     1 E14.7)
      N=2*(NBUSM+NLINEM)
C
C BUSV = BUS NUMBERS OF VOLTAGE MEASUREMENTS
C INCOV(N+K) = INVERSE VARIANCE OF EACH BUS VOLTAGE MEAS.
C
      READ(5,7) (BUSV(K),INCOV(N+K),K=1,NVOLTM)
    7 FORMAT(T5,I3,T16,E10.3)
      WRITE(6,8)
    8 FORMAT('0',7(2X,'BUS',2X,'VOLT INCOV'))
      WRITE(6,9) (BUSV(K),       INCOV(N+K),K=1,NVOLTM)
    9 FORMAT('0',7(2X,I3,2X,F10.3))
C
C VOLT,ANGLE = INITIAL ESTIMATE OF BUS VOLTAGES AND PHASE ANGLES
C
      READ(5,10) (VOLT(K),ANGLE(K),K=1,NBUS)
   10 FORMAT(7X,F7.4,2X,F7.4)
      WRITE(6,11)
   11 FORMAT('0',2X,'INITIAL VOLTAGES AND PHASE ANGLES')
      WRITE(6,50)
   50 FORMAT('0',5(2X,'BUS',2X,'VOLT MAG',2X,'PHASE ANG'))
      WRITE(6,51) (K,VOLT(K),ANGLE(K),K=1,NBUS)
   51 FORMAT('0',5(2X,I3,2X,F8.4,2X,F9.4))
      READ(5,875) (SNCON(K),ENCON(K),K=1,NBUS)
  875 FORMAT(T5,I3,T10,I3)
  901 CONTINUE
C
C CLINE = LIST OF LINES CONNECTED TO EACH BUS
C CBUS = LIST OF BUSES AT OPPOSITE END OF EACH CLINE
```

```
C SNCON = FIRST ELEMENT IN CLINE LIST FOR EACH BUS
C ENCON = LAST ELEMENT IN CLINE LIST FOR EACH BUS
C
      DO 928 K=1,NBUS
      M=SNCON(K)
      N=ENCON(K)
      READ(5,929) (CBUS(L),L=M,N)
  929 FORMAT(T15,10(I3,2X))
      READ(5,876) (CLINE(J),J=M,N)
  876 FORMAT(T15,10(I3,2X))
  928 CONTINUE
C
C NSTATE = NO. OF STATES
C NMEAS = NO. OF MEASUREMENTS
C LOOK=LIST OF STATES AS THEY APPEAR IN THE EQUATION FOR EACH MEASUREMENT
C LOOK1=LOCATION OF FIRST ELEMENT IN LOOK LIST FOR EACH MEAS.
C LOOK2=LOCATION OF LAST ELEMENT IN LOOK LIST FOR EACH MEAS.
C CODE = LIST OF MEASUREMENTS AS THEY INCLUDE EACH OF THE STATES
C CODE1 = LOCATION OF FIRST ELEMENT IN CODE LIST FOR EACH STATE
C CODE2 = LOCATION OF LAST ELEMENT IN CODE LIST FOR EACH STATE
C CSTATE: FOR EACH STATE, CSTATE LISTS THE HIGHER NUMBERED STATES HAVING CODE
C ELEMENTS IN COMMON WITH THAT STATE
C FCOM: FOR EACH STATE, FCOM LISTS THE LOCATION OF ELEMENTS IN THE CODE LIST
C THAT ARE COMMON TO THE CODE LIST ELEMENTS LISTED UNDER HIGHER NUMBERED STATES
C SCOM: SCOM LISTS THE STORAGE LOCATION OF THOSE ELEMENTS IN CODE WHOSE
C LOCATIONS WERE LISTED IN FCOM
C COM1 = LOCATION OF THE FIRST ELEMENT IN FCOM AND SCOM LISTS FOR EACH CSTATE
C COM2 = LOCATION OF THE LAST ELEMENT IN FCOM AND SCOM LISTS FOR EACH CSTATE
C NCOM1 = LOCATION OF FIRST ELEMENT IN CSTATE LIST FOR EACH STATE
C NCOM2 = LOCATION OF LAST ELEMENT IN CSTATE LIST FOR EACH STATE
C BCOM: FOR EACH STATE, BCOM LISTS THE LOWER NUMBERED STATES HAVING CODE
C ELEMENTS IN COMMON WITH THE CODE ELEMENTS FOR THAT STATE
C LBCOM LISTS THE LOCATION OF THE BCOM ELEMENTS IN THE CSTATE LIST
C SBCOM LISTS THE LOCATION OF THE FIRST ELEMENT IN THE BCOM AND LBCOM LISTS FOR
C EACH STATE
C EBCOM LISTS THE LOCATION OF THE LAST ELEMENT IN THE BCOM AND LBCOM LISTS FOR
C EACH STATE
```

```
C COL LISTS THE COLUMNS FOR THE ELEMENTS OF EACH ROW OF THE COEFFICIENT MATRIX
C THAT WILL BE GENERATED IN THIS PROGRAM
C SCOL = LOCATION OF THE FIRST ELEMENT IN COL FOR EACH STATE
C ECOL = LOCATION OF THE LAST ELEMENT IN COL FOR EACH STATE
C NELEM = NO. OF OFF-DIAGONAL ELEMENTS IN THE COEFFICIENT MATRIX THAT WILL BE
C GENERATED IN THIS PROGRAM
C
      NSTATE=2*NBUS-1
      NSM=NSTATE-1
      NMEAS=2*(NBUSM+NLINEM)+NVOLTM
      READ(5,813) (LOOK1(K),LOOK2(K),K=1,NMEAS)
  813 FORMAT(10(I3,2X,I3))
      DO 814 K=1,NMEAS
      MA=LOOK1(K)
      MB=LOOK2(K)
      READ(5,815) (LOOK(L),L=MA,MB)
  815 FORMAT(20(I3,1X))
  814 CONTINUE
      NSM=NSTATE-1
      READ(5,821) (CODE1(K),CODE2(K),NCOM1(K),NCOM2(K),SBCOM(K),
     1 EBCOM(K),K=1,NSM)
  821 FORMAT(I3,T6,I3,T11,I3,T16,I3,T21,I3,T26,I3)
      READ(5,838) CODE1(NSTATE),CODE2(NSTATE),SBCOM(NSTATE),
     1 EBCOM(NSTATE)
  838 FORMAT(I3,T6,I3,T21,I3,T26,I3)
      DO 819 K=1,NSM
      MA=CODE1(K)
      MB=CODE2(K)
      READ(5,820) (CODE(L),L=MA,MB)
  820 FORMAT(20(I3,1X))
      NA=NCOM1(K)
      NB=NCOM2(K)
      IF(NA.LT.1) GO TO 819
      READ(5,822) (COM1(L),COM2(L),CSTATE(L),L=NA,NB)
  822 FORMAT(16(I4,1X))
      DO 823 J=NA,NB
      IA=COM1(J)
```

```
      IB=COM2(J)
      READ(5,824) (FCOM(L),SCOM(L),L=IA,IB)
  824 FORMAT(20(I3,1X))
  823 CONTINUE
  819 CONTINUE
      DO 590 K=2,NSTATE
      KK=SBCOM(K)
      LL=EBCOM(K)
      IF(KK.LT.1) GO TO 590
      READ(5,591) (BCOM(L),LBCOM(L),L=KK,LL)
  591 FORMAT(16(I4,1X))
  590 CONTINUE
      JA=CODE1(NSTATE)
      JB=CODE2(NSTATE)
      READ(5,839) (CODE(L),L=JA,JB)
  839 FORMAT(20(I3,1X))
      READ(5,580) NELEM
  580 FORMAT(I4)
      WRITE(6,581) NELEM
  581 FORMAT('0',1X,'NELEM=',I4)
      READ(5,582) (COL(K),K=1,NELEM)
  582 FORMAT(20(I3,1X))
      READ(5,490) (SCOL(K),ECOL(K),K=1,NSTATE)
  490 FORMAT(15(I4,1X))
  902 CONTINUE
      CALL CMEAS
      N=2*(NBLSM+NLINEM)
      M=N+1
      IF(NMEAS.LT.M) GO TO 837
C
C F = JACOBIAN MATRIX
C
      DO 836 K=M,NMEAS
      F(LOCK1(K))=1.0
  836 CONTINUE
  837 CONTINUE
C
```

```
C COST(1)= COST FUNCTICN FOR PREVIOUS ITERATION
C COST(2)= COST FUNCTICN FOR LATEST ITERATION
C
      CCST(2)=0.0
C
C ICOUNT = ITERATICN COUNTER AND TRIP CARD. THIS IS SET=0 EACH TIME A MEASURE-
C MENT SET IS READ. ICCUNT=1 IS READ TO TERMINATE PROGRAM.
C
   30 READ(5,12) ICCUNT
   12 FORMAT(I3)
      IF(ICOUNT.GT.0) GO TO 200
      INSET=INSET+1
      WRITE(6,13) INSET
   13 FORMAT('0','INPUT DATA SET=',I3)
C
C Z = SET CF MEASUREMENT VALUES. THESE ARE ALWAYS PROCESSED IN THE FOLLOWING
C ORDER. REAL BUS INJECTIONS, REACTIVE BUS INJECTIONS, REAL LINE FLOWS, REACTIVE
C LINE FLOWS, BUS VOLTAGES. ALL BUS INJ. AND LINE FLCWS MUST INCLUDE BOTH REAL
C AND REACTIVE PARTS.
C
      READ(5,14) (Z(K),Z(NBUSM+K),K=1,NBUSM)
   14 FCRMAT(T11,E10.3,T31,E10.3)
      WRITE(6,31)
   31 FORMAT('0',T2,'BUS',T8,'REAL PWR',T20,'IMAG PWR',T32,'BUS',T38,
     1'REAL PWR',T50,'IMAG PWR',T62,'BUS',T68,'REAL PWR',T80,'IMAG PWR',
     1T92,'BUS',T98,'REAL PWR',T110,'IMAG PWR')
      WRITE(6,32) (BUS(K),Z(K),Z(NBUSM+K),K=1,NBUSM)
   32 FORMAT('0',T2,I3,T8,E10.3,T20,E10.3,T32,I3,T38,E10.3,T50,E10.3,
     1 T62,I3,T68,E10.3,T80,E10.3,T92,I3,T98,E10.3,T110,E10.3)
      N=2*NBUSM
      M=N+1
      I=N+NLINEM
      READ(5,15) (Z(K),Z(K+NLINEM),K=M,I)
   15 FCRMAT(T11,E10.3,T31,E10.3)
      WRITE(6,33)
   33 FORMAT('0',T2,'LINE',T8,'REAL PWR',T20,'IMAG PWR',T32,'LINE',T38,
     1'REAL PWR',T50,'IMAG PWR',T62,'LINE',T68,'REAL PWR',T80,'IMAG ',
```

```
     1'PWR',T92,'LINE',T98,'REAL PWR',T110,'IMAG PWR')
      WRITE(6,32) (LINE(K-N),Z(K),Z(K+NLINEM),K=M,I)
      N=2*(NBUSM+NLINEM)
      M=N+1
      I=N+NVOLTM
      READ(5,16) (Z(K),K=M,I)
   16 FORMAT(T11,E10.3)
      WRITE(6,34)
   34 FORMAT('0',T2,'BUS',T8,'VOLT MAG',T20 BUS',T26,'VOLT MAG',T38,
     1 'BUS',T44,'VOLT MAG',T56,'BUS',T62,'VOLT MAG',T74,'BUS',T80,
     1 'VOLT MAG',T92,'BUS',T98,'VOLT MAG',T110,'BUS',T116,'VOLT MAG')
      WRITE(6,35) (BUSV(K),Z(N+K),K=1,NVOLTM)
   35 FORMAT('0',T2,I3,T8,E10.3,T20,I3,T26,E10.3,T38,I3,T44,E10.3,T56,
     1I3,T62,E10.3,T74,I3,T80,E10.3,T92,I3,T98,E10.3,T110,I3,T116,E10.3)
   80 CONTINUE
      CALL STOPTM(T)
      WRITE(6,980) T
  980 FORMAT('0',2X,'T=',F8.3)
      T=0.0
      CALL STARTM(T)
      COST(1)=COST(2)
      COST(2)=0.0
      N=2*(NBUSM+NLINEM)+NVOLTM
C
C C = VALUES OF THE MEASUREMENTS CALCULATED FROM THE LATEST ESTIMATES OF VOLT
C AND ANGLE. THESE ARE PROCESSED IN THE SAME ORDER AS Z IN SUBROUTINE CMEAS.
C
      DO 81 K=1,N
      COST(2)=COST(2)+INCOV(K)*(Z(K)-C(K))*(Z(K)-C(K))
   81 CONTINUE
      WRITE(6,17) COST(2)
   17 FORMAT('0',2X,'COST=',E10.3)
C
C DIFF = CONVERGENCE TOLERANCE
C
      DIFF= ABS(COST(2)-COST(1))
      WRITE(6,18) DIFF
```

```
   18 FORMAT('0',2X,'DIFFERENCE=',E10.3)
  903 CONTINUE
      IF(DIFF.GT.0.005*COST(2).AND.DIFF.GT.0.000003) GO TO 635
C
C NF = NO. OF ELEMENTS IN JACOBIAN MATRIX, F
C
  636 NF = LOOK2(NMEAS)
      WRITE(7,600) (F(K),K=1,NF)
  600 FORMAT(5(2X,E14.7))
      WRITE(7,601) (C(K),K=1,NMEAS)
  601 FORMAT(5(2X,E14.7))
      NBUS1=NBUS-1
C
C VOLT,ANGLE = NEW STATE ESTIMATE
C
      WRITE(7,602) (VOLT(K),K=1,NBUS)
      WRITE(7,602)                          (ANGLE(L),L=1,NBUS1)
  602 FORMAT(5(2X,E14.7))
      GO TO 30
  635 IF(ICOUNT.GE.2) GO TO 200
      ICOUNT=ICOUNT+1
      WRITE(6,19) ICOUNT
   19 FORMAT('0',2X,'ITERATION COUNT=',I3)
      CALL JACOB
      CALL PREMAT
       CALL STOPTM(T)
      WRITE(6,981) T
  981 FORMAT('0',2X,'T=',F8.3)
      T=0.0
      CALL STARTM(T)
      CALL SCLMAT
      CALL CMEAS
      GO TO 80
  200 CONTINUE
      STOP
      END
```

```
      SUBROUTINE CMEAS
C
C THIS SUBROUTINE CALCULATES THE VALUES OF THE MEASUREMENTS FROM THE STATE
C ESTIMATES
C ALL ARRAYS ARE DEFINED IN MAIN PROGRAM
C
      REAL              VOLT(58),SUMRY(58),SUMIY(58),YMAG(70),YANG(70),
     1 ANGLE(58),RY(70),IY(70)
      REAL*8 C(175)
      INTEGER*2 BUS(60),LINE(60),HEADY(70),TAILY(70),HEAD(60),TAIL(60),
     1 BUSV(60),SNCON(60),ENCON(60),CLINE(140),CBUS(140)
      COMMON/ONE/C,NBUSM,VOLT,BUS,SUMRY,SUMIY,NLINE,YMAG,YANG,ANGLE,
     1 NLINEM,RY,IY,NVOLTM,HEADY,TAILY,HEAD,TAIL,LINE,BUSV,SNCON,ENCON,
     1 CLINE,CBUS
      WRITE(6,925)
  925 FORMAT('0',2X,'CALCULATED VALUES OF MEASUREMENTS')
      WRITE(6,170)
  170 FORMAT('0',T3,'BUS',T13,'REAL PWR',T25,'IMAG PWR')
      DO 160 K=1,NBUSM
      KBUS=BUS(K)
      KBM=K+NBUSM
      VB=VOLT(KBUS)
      VBS=VB*VB
      C(K)=VBS*SUMRY(KBUS)
      C(KBM)=VBS*SUMIY(KBUS)
      LL=SNCON(KBUS)
      KK=ENCON(KBUS)
      DO 161 L=LL,KK
      LCL=CLINE(L)
      LCB=CBUS(L)
      AA=YANG(LCL)+ANGLE(KBUS)-ANGLE(LCB)
      BB=VB*VOLT(LCB)*YMAG(LCL)
      C(K)=C(K)+BB*COS(AA)
      C(KBM)=C(KBM)+BB*SIN(AA)
  161 CONTINUE
      IJ=NBUSM+K
      WRITE(6,162) KBUS,C(K),C(IJ)
```

```
162 FORMAT('0',T3,I3,T14,F8.4,T26,F8.4)
160 CONTINUE
    WRITE(6,172)
172 FORMAT('0',T3,'LINE',T10,'HEAD',T17,'TAIL',T24,'RE LINE PWR',T39,
   1'IM LINE PWR')
    N=2*NBUSM
    M=N+NLINEM
    DO 165 K=1,NLINEM
    KHEAD=HEAD(K)
    KTAIL=TAIL(K)
    LINEK=LINE(K)
    VH=VOLT(KHEAD)
    A=VH*VOLT(KTAIL)*YMAG(LINEK)
    B=ANGLE(KHEAD)-ANGLE(KTAIL)+YANG(LINEK)
    D=VH*VH
    IL=N+K
    IK=M+K
    C(IL)=A*COS(B)+D*RY(K)
    C(IK)=A*SIN(B)+D*IY(K)
    WRITE(6,173) LINEK,KHEAD,KTAIL,C(IL),C(IK)
173 FORMAT('0',T3,I3,T10,I3,T17,I3,T24,F8.4,T39,F8.4)
165 CONTINUE
    WRITE(6,174)
174 FORMAT('0',T3,'BUS',T13,'VOLT MAG')
    N=2*(NBUSM+NLINEM)
    DO 166 K=1,NVOLTM
    IL=N+K
    KBUSV=BUSV(K)
    C(IL)=VOLT(KBUSV)
    WRITE(6,175) KBUSV,C(IL)
175 FORMAT('0',T3,I3,T13,F7.3)
166 CONTINUE
    RETURN
    END
```

```
      SUBROUTINE JACOB
C
C THIS SUBROUTINE CALCULATES THE JACOBIAN MATRIX
C ALL ARRAYS ARE DEFINED IN MAIN PROGRAM
C
      REAL INCCV(175),YMAG(70),YANG(70),SUMRY(58),SUMIY(58),RY(70),
     1 IY(70),Z(175),        VOLT(58),        ANGLE(58),F(750),
     1 TEMPR(750)
      REAL   ELEM(1200),GAIN(5500),COST(2)
      REAL*8                DF,DF1,DF2,DF3,AC,CV,DD,EE,SRH,
     1 TGAIN,C(175),SUB,RHS(120),EWORK(120),FWORK(120),AEWORK,GWORK,
     1TDIAG,DIAG(120),AFWORK,DIAGK,RHSK,RHSL,ADIAG
      INTEGER*2 HEADY(70),COL(1200),SGAIN(120),EGAIN(120),SCOL(120),
     1 TAILY(70),HEAD(60),TAIL(60),BUS(60),LINE(60),BUSV(60),SNCON(60),
     1 ENCON(60),CLINE(140),CBUS(140),LOOK1(175),LOOK2(175),LOOK(750),
     1 CODE1(120),CODE2(120),CODE(750),NCOM1(120),NCOM2(120),COM1(600),
     1 COM2(600),FCOM(1600),SCOM(1600),LIL(120),CSTATE(600),SBCOM(120),
     1 EBCOM(120),BCOM(600),LBCOM(600),ECOL(120),CEWORK(120),CFWORK(120)
      INTEGER*2 COLG(5500)
      COMMON/ONE/C,NBUSM,VOLT,BUS,SUMRY,SUMIY,NLINE,YMAG,YANG,ANGLE,
     1 NLINEM,RY,IY,NVOLTM,HEADY,TAILY,HEAD,TAIL,LINE,BUSV,SNCON,ENCON,
     1 CLINE,CBUS
      COMMON/TWO/DF,DF1,DF2,DF3,AC,CV,DD,EE,SRH,TGAIN,SUB,RHS,EWORK,
     1 FWORK,AEWORK,GWORK,TDIAG,DIAG,AFWORK,DIAGK,RHSK,RHSL,ADIAG,
     1 INCCV,Z,F,TEMPR,ELEM,GAIN,NBUS,NMEAS,NSTATE,NSM,COL,SGAIN,EGAIN,
     1 SCOL,LOOK1,LOOK2,LOOK,CODE1,CODE2,CODE,NCOM1,NCOM2,COM1,COM2,
     1 FCOM,SCOM,LIL,CSTATE,SBCOM,EBCOM,BCOM,LBCOM,ECOL,CEWORK,CFWORK,
     1 COLG
      DO 84 K=1,NBUSM
      KBUS=BUS(K)
      BVOLT=2*VOLT(KBUS)
      N=SNCON(KBUS)
      M=ENCON(KBUS)
      KOD=LOOK1(K)
      KOD1=LOOK1(K+NBUSM)
      DF    =BVOLT*SUMRY(KBUS)
      DF1    =BVOLT*SUMIY(KBUS)
```

```
C
C LKT=INDEX FOR STORING ELEMENTS OF F
C
      LKT=0
      IF(KBUS.EQ.NBUS) GO TO 85
      LKT=LKT+1
      DF2=0.0
      DF3=0.0
      DO 86 L=N,M
      LKT=LKT+1
      LCL=CLINE(L)
      LCB=CBUS(L)
      YM=YMAG(LCL)
      CVOLT=VOLT(LCB)
      BBVOLT=BVOLT/2.0
      BB=YANG(LCL)+ANGLE(KBUS)-ANGLE(LCB)
      AA=YM*COS(BB)
      CC=YM*SIN(BB)
      AC=CVOLT*AA
      AB=BBVOLT*AA
      CV=CVOLT*CC
      BV=BBVOLT*CC
      DF=DF+AC
      F(KOD+LKT)=AB
      DF1=DF1+CV
      F(KOD1+LKT)=BV
      IF(LCB.EQ.NBUS) GO TO 87
      LKT=LKT+1
      F(KOD +LKT)=BBVOLT*CV
      DF2=DF2-BBVOLT*CV
      F(KOD1 + LKT)=-BBVOLT*AC
      DF3=DF3+BBVOLT*AC
      GO TO 86
   87 DF2=DF2-BBVOLT*CV
      DF3=DF3+BBVOLT*AC
   86 CONTINUE
      F(KOD)=DF
```

```
      F(KOD1)=DF1
      F(KOD+1)=DF2
      F(KOD1+1)=DF3
      GO TO 84
   85 DO 88 L=N,M
      LKT=LKT+1
      LCL=CLINE(L)
      LCB=CBUS(L)
      YM=YMAG(LCL)
      CVOLT=VOLT(LCB)
      BBVOLT=BVOLT/2.0
      BB=YANG(LCL)+ANGLE(KBUS)-ANGLE(LCB)
      AA=YM*COS(BB)
      CC=YM*SIN(BB)
      AC=CVOLT*AA
      AB=BBVOLT*AA
      CV=CVOLT*CC
      BV=BBVOLT*CC
      DF=DF+AC
      F(KOD +LKT)=AB
      DF1=DF1+CV
      F(KOD1+LKT)=BV
      LKT=LKT+1
      F(KOD+LKT)=BBVOLT*CV
      F(KOD1+LKT)=-BBVOLT*AC
   88 CONTINUE
      F(KOD)=DF
      F(KOD1)=DF1
   84 CONTINUE
      NN=NBUS+1
      MM=2*NBUS-1
      N=NBUSM+1
      M=2*NBUSM
      DO 97 K=1,NLINEM
      KLINE=LINE(K)
      KHEAD=HEAD(K)
      KTAIL=TAIL(K)
```

```
      KNB=2*NBUSM+K
      KNBL=KNB+NLINEM
      YM=YMAG(KLINE)
      VT=VOLT(KTAIL)
      VH=VOLT(KHEAD)
      AA=YANG(KLINE)+ANGLE(KHEAD)-ANGLE(KTAIL)
      BB=YM*COS(AA)
      CC=YM*SIN(AA)
      DD=VT*BB
      EE=VT*CC
      FF=2*VH
      GG=VH*EE
      HH=VH*DD
      KKK=LOCK1(KNB)
      LLL=LOCK1(KNBL)
      DF=DD+FF*RY(K)
      F(KKK)=DF
      DF1=EE+FF*IY(K)
      F(LLL)=DF1
      IF(KHEAD.EQ.NBUS) GO TO 98
      F(KKK+2)=VH*BB
      F(LLL+2)=VH*CC
      F(KKK+1)=-GG
      F(LLL+1)=HH
      IF(KTAIL.EQ.NBUS) GO TO 97
      F(KKK+3)=GG
      F(LLL+3)=-HH
      GO TO 97
   98 F(KKK+1)=VH*BB
      F(LLL+1)=VH*CC
      F(KKK+2)=GG
      F(LLL+2)=-HH
   97 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE PREMAT
C
C THIS SUBROUTINE CALCULATES THE MATRIX PRODUCTS NECESSARY FOR FINDING THE
C STATE ESTIMATE
C ARRAYS NOT DEFINED HERE ARE DEFINED IN MAIN PROGRAM
C
      REAL INCOV(175),YMAG(70),YANG(70),SUMRY(58),SUMIY(58),RY(70),
     1 IY(70),Z(175),       VOLT(58),        ANGLE(58),F(750),
     1 TEMPR(750)
      REAL   ELEM(1200),GAIN(5500),COST(2)
      REAL*8                 DF,DF1,DF2,DF3,AC,CV,DD,EE,SRH,
     1 TGAIN,C(175),SUB,RHS(120),EWORK(120),FWORK(120),AEWORK,GWORK,
     1TDIAG,DIAG(120),AFWORK,DIAGK,RHSK,RHSL,ADIAG
      INTEGER*2 HEADY(70),COL(1200),SGAIN(120),EGAIN(120),SCOL(120),
     1 TAILY(70),HEAD(60),TAIL(60),BUS(60),LINE(60),BUSV(60),SNCON(60),
     1 ENCON(60),CLINE(140),CBUS(140),LOOK1(175),LOOK2(175),LOOK(750),
     1 CODE1(120),CODE2(120),CODE(750),NCOM1(120),NCOM2(120),COM1(600),
     1 COM2(600),FCOM(1600),SCOM(1600),LIL(120),CSTATE(600),SBCOM(120),
     1 EBCOM(120),BCOM(600),LBCOM(600),ECOL(120),CEWORK(120),CFWORK(120)
      INTEGER*2 COLG(5500)
      COMMON/ONE/C,NBUSM,VOLT,BUS,SUMRY,SUMIY,NLINE,YMAG,YANG,ANGLE,
     1 NLINEM,RY,IY,NVOLTM,HEADY,TAILY,HEAD,TAIL,LINE,BUSV,SNCON,ENCON,
     1 CLINE,CBUS
      COMMON/TWO/DF,DF1,DF2,DF3,AC,CV,DD,EE,SRH,TGAIN,SUB,RHS,EWORK,
     1 FWORK,AEWORK,GWORK,TDIAG,DIAG,AFWORK,DIAGK,RHSK,RHSL,ADIAG,
     1 INCOV,Z,F,TEMPR,ELEM,GAIN,NBUS,NMEAS,NSTATE,NSM,COL,SGAIN,EGAIN,
     1 SCOL,LOOK1,LOOK2,LOOK,CODE1,CODE2,CODE,NCOM1,NCOM2,COM1,COM2,
     1 FCOM,SCOM,LIL,CSTATE,SBCOM,EBCOM,BCOM,LBCOM,ECOL,CEWORK,CFWORK,
     1 COLG
      DO 800 K=1,NMEAS
      I=LOOK1(K)
      J=LOOK2(K)
      STORE=INCOV(K)
      SUB=Z(K)-C(K)
C
C GAIN = MATRIX PRODUCT, F'*INCOV
C TEMPR = INTERMEDIATE ARRAY FOR FINDING F'*INCOV*(Z-C), (SUMMING TO GET EACH
```

```
C ROW TERM HAS NOT YET BEEN PERFORMED)
C
      DO 801 L=I,J
      GAIN(L)=F(L)*STORE
      TEMPR(L)=GAIN(L)*SUB
  801 CONTINUE
  800 CONTINUE
C
C RHS = VECTOR, F'*INCOV*(Z-C)
C NELMT = NO. OF ELEMENTS IN F
C
      DO 802 K=1,NSTATE
      RHS(K)=0.0
  802 CONTINUE
      NELMT=LOOK2(NMEAS)
      DO 803 K=1,NELMT
      L=LOOK(K)
      RHS(L)=RHS(L)+TEMPR(K)
  803 CONTINUE
C
C LIL = INDEX ARRAY USED FOR TRANSPOSING GAIN AND F
C ELEM = F'
C TEMPR = GAIN'
C
      DO 804 K=1,NSTATE
      LIL(K)=0
  804 CONTINUE
      DO 805 K=1,NELMT
      I=LOOK(K)
      J=CODE1(I)+LIL(I)
      ELEM(J)=F(K)
      TEMPR(J)=GAIN(K)
      LIL(I)=LIL(I)+1
  805 CONTINUE
C
C TDIAG = DIAG(K) = K TH DIAGONAL OF COEFFICIENT MATRIX, (F'*INCOV*F)
C
```

```
      DO 806 K=1,NSM
      I=CODE1(K)
      J=CODE2(K)
      II=NCOM1(K)
      JJ=NCOM2(K)
      TDIAG=0.0
      DO 807 L=I,J
      TDIAG=TDIAG+ELEM(L)*TEMPR(L)
  807 CONTINUE
C
C TGAIN = GAIN(KK) = KK TH UPPER OFF-DIAGONAL TERM OF THE COEFFICIENT MATRIX,
C (F'*INCOV*F)
C
      DIAG(K)=TDIAG
      IF(II.EQ.0) GO TO 806
      DO 808 KK=II,JJ
      KCM1=COM1(KK)
      KCM2=COM2(KK)
      TGAIN=0.0
      DO 809 LL=KCM1,KCM2
      TGAIN=TGAIN+ELEM(FCOM(LL))*TEMPR(SCOM(LL))
  809 CONTINUE
      GAIN(KK)=TGAIN
  808 CONTINUE
  806 CONTINUE
      I=CODE1(NSTATE)
      J=CODE2(NSTATE)
      TDIAG=0.0
      DO 180 K=I,J
      TDIAG=TDIAG+ELEM(K)*TEMPR(K)
  180 CONTINUE
      DIAG(NSTATE)=TDIAG
      MMM=NCOM1(1)
       M =NCOM2(1)
      IF(MMM.LT.1) GO TO 898
C
C ELEM: INCLUDES ALL OFF-DIAGONAL TERMS OF THE COEFFICIENT MATRIX, (F'*INCOV*F)
```

```
C
      DC 550 K=MMM,M
      ELEM(K)=GAIN(K)
  550 CONTINUE
  898 CCNTINLE
      DO 551 K=2,NSM
      KSBCCM=SBCOM(K)
      KEBCOM=EBCOM(K)
      IF(KSBCCM.LT.1) GO TO 553
      DO 552 L=KSBCOM,KEBCOM
      M=M+1
      ELEM(M)=GAIN(LBCOM(L))
  552 CCNTINUE
  553 CONTINUE
      KNCOM1=NCOM1(K)
      KNCOM2=NCOM2(K)
      IF(KNCOM1.LT.1) GO TO 551
      DO 554 L=KNCOM1,KNCOM2
      M=M+1
      ELEM(M)=GAIN(L)
  554 CCNTINUE
  551 CONTINUE
      NSBCOM=SBCOM(NSTATE)
      NEBCCM=EBCOM(NSTATE)
      IF(NSBCCM.LT.1) GO TO 555
      DO 556 K=NSBCCM,NEBCOM
      M=M+1
      ELEM(M)=GAIN(LBCCM(K))
  556 CONTINUE
  555 CCNTINUE
      RETURN
      END


      SLBROUTINE SOLMAT
C
```

```
C THIS SUBROUTINE TRIANGULARIZES THE COEFFICIENT MATRIX, (F'*INCOV*F), AND
C CALCULATES THE NEW ESTIMATE OF VOLT AND ANGLE BY BACK SUBSTITUTION
C ARRAYS NOT DEFINED HERE ARE DEFINED IN MAIN PROGRAM OR PREMAT
C
      REAL INCOV(175),YMAG(70),YANG(70),SUMRY(58),SUMIY(58),RY(70),
     1 IY(70),Z(175),        VOLT(58),          ANGLE(58),F(750),
     1 TEMPR(750)
      REAL   ELEM(1200),GAIN(5500),COST(2)
      REAL*8                DF,DF1,DF2,DF3,AC,CV,DD,EE,SRH,
     1 TGAIN,C(175),SUB,RHS(120),EWORK(120),FWORK(120),AEWORK,GWORK,
     1TDIAG,DIAG(120),AFWORK,DIAGK,RHSK,RHSL,ADIAG
      INTEGER*2 HEADY(70),COL(1200),SGAIN(120),EGAIN(120),SCOL(120),
     1 TAILY(70),HEAD(60),TAIL(60),BUS(60),LINE(60),BUSV(60),SNCON(60),
     1 ENCON(60),CLINE(140),CBUS(140),LOOK1(175),LOOK2(175),LOOK(750),
     1 CODE1(120),CODE2(120),CODE(750),NCOM1(120),NCOM2(120),COM1(600),
     1 COM2(600),FCOM(1600),SCOM(1600),LIL(120),CSTATE(600),SBCOM(120),
     1 EBCOM(120),BCOM(600),LBCOM(600),ECOL(120),CEWORK(120),CFWORK(120)
      INTEGER*2 COLG(5500)
      COMMON/ONE/C,NBUSM,VOLT,BUS,SUMRY,SUMIY,NLINE,YMAG,YANG,ANGLE,
     1 NLINEM,RY,IY,NVOLTM,HEADY,TAILY,HEAD,TAIL,LINE,BUSV,SNCON,ENCON,
     1 CLINE,CBUS
      COMMON/TWO/DF,DF1,DF2,DF3,AC,CV,DD,EE,SRH,TGAIN,SUB,RHS,EWORK,
     1 FWORK,AEWORK,GWORK,TDIAG,DIAG,AFWORK,DIAGK,RHSK,RHSL,ADIAG,
     1 INCOV,Z,F,TEMPR,ELEM,GAIN,NBUS,NMEAS,NSTATE,NSM,COL,SGAIN,EGAIN,
     1 SCOL,LOOK1,LOOK2,LOOK,CODE1,CODE2,CODE,NCOM1,NCOM2,COM1,COM2,
     1 FCOM,SCOM,LIL,CSTATE,SBCOM,EBCOM,BCOM,LBCOM,ECOL,CEWORK,CFWORK,
     1 COLG
C
C GAIN = (F'*INCOV*F) AFTER GAUSSIAN ELIMINATION
C SGAIN = LOCATION OF THE FIRST ELEMENT IN EACH ROW OF GAIN
C EGAIN = LOCATION OF THE LAST ELEMENT IN EACH ROW OF GAIN
C COLG = COLUMN NUMBER OF EACH ELEMENT IN GAIN
C
       SGAIN(1)=1
       EGAIN(1)=ECOL(1)
       ADIAG=1/DIAG(1)
       RHS(1)=RHS(1)*ADIAG
```

```
      IND=EGAIN(1)
      DO 401 K=1,IND
      GAIN(K)=ELEM(K)*ADIAG
      COLG(K)=COL(K)
  401 CONTINUE
C
C EWORK = WORKING ROW USED TO ELIMINATE TERMS TO THE LEFT OF THE DIAGONAL IN
C EACH ROW OF ELEM
C FWORK: SIMILAR TO EWORK, ELEMENTS ARE PASSED BACK AND FORTH BETWEEN EWORK AND
C FWORK AS EACH ELEMENT ON THE LEFT IS ELIMINATED
C CEWORK = COLUMN NUMBER FOR EACH ELEMENT OF EWORK
C CFWORK = COLUMN NUMBER FOR EACH ELEMENT OF FWORK, THESE ELEMENTS ARE ALSO
C PASSED BETWEEN CEWORK AND CFWORK AS THE ELIMINATION PROGRESSES
C KJ = INDEX FOR STORING THE ELEMENTS OF EWORK AND CEWORK
C KS = INDEX FOR STORING THE ELEMENTS OF FWORK AND CFWORK
C
      DO 405 K=2,NSTATE
      KJ=0
      KSCOL=SCOL(K)
      KECOL=ECOL(K)
      DO 402 KI=KSCOL,KECOL
      KJ=KJ+1
      EWORK(KJ)=ELEM(KI)
      CEWORK(KJ)=COL(KI)
  402 CONTINUE
C
C KT = COUNTER USED TO PREVENT DUPLICATION OF ELEMENTS WHEN TWO ROWS ARE ADDED
C KLAD = SIGNAL USED TO PREVENT DUPLICATION
C GWORK = THE MULTIPLIED ELEMENT FROM A GAIN ROW THAT IS ADDED TO EWORK OR FWORK
C KX = COUNTER USED TO PREVENT DUPLICATION
C LX: USED TO RECORD DO LOOP INDEX
C
      K1=K-1
      RHSK=RHS(K)
      DIAGK=DIAG(K)
      DO 406 L=1,NSTATE
      KCE1=CEWORK(1)
```

```
      IF(KCE1.GT.K) GO TO 450
      LSE=SGAIN(KCE1)
      LEE=EGAIN(KCE1)
      AEWORK=EWORK(1)
      RHSL=RHS(KCE1)
      RHSK=RHSK-RHSL*AEWORK
      KT=2
      KS=0
      KLAD=0
      IF(LSE.GT.LEE) GO TO 866
      DO 403 LT=LSE,LEE
      GWORK=GAIN(LT)*AEWORK
      KCOLG=COLG(LT)
      IF(K.EQ.KCOLG) GO TO 412
      KX=1
      IF(KT.GT.KJ) GO TO 410
  732 CONTINUE
      DO 408 KB=KT,KJ
      LX=KB
      IF(CEWORK(KB).EQ.KCOLG) GO TO 409
      IF(CEWORK(KB).GT.KCOLG.AND.KLAD.LT.1) GO TO 410
      KS=KS+1
      FWORK(KS)=EWORK(KB)
      CFWORK(KS)=CEWORK(KB)
      KX=KX+1
  408 CONTINUE
      IF(KLAD.GE.1) GO TO 403
      KT=KT+KX-1
      KS=KS+1
      FWORK(KS)=-GWORK
      CFWORK(KS)=KCOLG
      GO TO 403
  409 KS=KS+1
      FWORK(KS)=EWORK(LX)-GWORK
      CFWORK(KS)=KCOLG
      KT=KT+KX
      IF(LT.GE.LEE.AND.KT.LE.KJ) GO TO 731
```

```
      GO TO 403
  410 KS=KS+1
      FWORK(KS)=-GWORK
      CFWORK(KS)=KCCLG
      KT=KT+KX-1
      IF(LT.GE.LEE.AND.KT.LE.KJ) GO TO 731
      GO TO 403
  731 KLAD=1
      GO TO 732
  412 DIAGK=DIAGK-GWORK
      IF(LT.GE.LEE.AND.KT.LE.KJ) GO TO 731
  403 CONTINUE
      GO TO 867
  866 IF(KJ.LE.1) GO TO 399
      DO 868 LY=2,KJ
      KS=KS+1
      FWORK(KS)=EWORK(LY)
      CFWORK(KS)=CEWORK(LY)
  868 CONTINUE
  867 IF(KS.LT.1) GO TO 399
      KCF1=CFWORK(1)
      IF(KCF1.GT.K) GO TO 399
      LSF=SGAIN(KCF1)
      LEF=EGAIN(KCF1)
      AFWORK=FWORK(1)
      RHSL=RHS(KCF1)
      RHSK=RHSK-RHSL*AFWORK
      KT=2
      KJ=0
      KLAD=0
      IF(LSF.GE.LEF) GO TO 869
      DO 433 LT=LSF,LEF
      GWORK=GAIN(LT)*AFWORK
      KCOLG=CCLG(LT)
      IF(K.EQ.KCOLG) GO TO 442
      KX=1
      IF(KT.GT.KS) GO TO 440
```

```
734 CCNTINUE
    DO 438 KB=KT,KS
    LX=KB
    IF(CFWORK(KB).EQ.KCOLG) GO TO 439
    IF(CFWORK(KB).GT.KCOLG.AND.KLAD.LT.1) GO TO 440
    KJ=KJ+1
    EWORK(KJ)=FWORK(KB)
    CEWORK(KJ)=CFWORK(KB)
    KX=KX+1
438 CCNTINUE
    IF(KLAD.GE.1) GO TO 433
    KT=KT+KX-1
    KJ=KJ+1
    EWORK(KJ)=-GWORK
    CEWORK(KJ)=KCCLG
    GC TC 433
439 KJ=KJ+1
    EWORK(KJ)=FWORK(LX)-GWORK
    CEWORK(KJ)=KCOLG
    KT=KT+KX
    IF(LT.GE.LEF.AND.KT.LE.KS) GO TO 733
    GO TO 433
440 KJ=KJ+1
    EWORK(KJ)=-GWORK
    CEWORK(KJ)=KCOLG
    KT=KT+KX-1
    IF(LT.GE.LEF.AND.KT.LE.KS) GO TO 733
    GC TO 433
733 KLAD=1
    GO TC 734
442 DIAGK=DIAGK-GWCRK
    IF(LT.GE.LEF.AND.KT.LE.KS) GO TO 733
433 CCNTINUE
    GO TO 87C
869 IF(KS.LE.1) GO TO 450
    DC 871 LY=2,KS
    KJ=KJ+1
```

```
      EWORK(KJ)=FWORK(LY)
      CEWORK(KJ)=CFWCRK(LY)
  871 CCNTINUE
  870 IF(KJ.LT.1) GO TO 450
  406 CCNTINUE
  399 ADIAG=1/DIAGK
C
C RHS IS USEC TC STORE THE CALCULATED CHANGE IN VOLT AND ANGLE
C
      RHS(K)=RHSK*ADIAG
      KEGAIN=EGAIN(K-1)
      EGAIN(K)=KEGAIN+KS
      IF(KS.LT.1) GO TC 405
      DO 451 M=1,KS
      GAIN(M+KEGAIN)=FWORK(M)*ADIAG
      COLG(M+KEGAIN)=CFWORK(M)
  451 CONTINUE
      GO TO 453
  450 ADIAG=1/DIAGK
      RHS(K)=RHSK*ADIAG
      KEGAIN=EGAIN(K-1)
      EGAIN(K)=KEGAIN+KJ
      IF(KJ.LT.1) GO TC 405
      DO 452 M=1,KJ
      GAIN(M+KEGAIN)=EWORK(M)*ADIAG
      COLG(M+KEGAIN)=CEWORK(M)
  452 CCNTINUE
  453 SGAIN(K)=KEGAIN+1
  405 CCNTINUE
      NS=NSTATE-1
      DC 470 K=1,NS
      KSTATE=NSTATE-K
      SRH=RHS(KSTATE)
      KK=SGAIN(KSTATE)
      LL=EGAIN(KSTATE)
      DO 471 L=KK,LL
      SRH=SRH-GAIN(L)*RHS(COLG(L))
```

```
471 CCNTINUE
    RHS(KSTATE)=SRH
470 CONTINUE
    NBUS1=NBUS-1
    DO 472 K=1,NBUS1
    VOLT(K)=RHS(K)+VOLT(K)
    ANGLE(K)=RHS(NBUS+K)+ANGLE(K)
472 CCNTINUE
    VCLT(NBUS)=RHS(NBUS)+VOLT(NBUS)
    WRITE(6,897) (VCLT(K),ANGLE(K),K=1,NBUS)
897 FCRMAT('0',8(2X,E11.4))
    RETURN
    END
```

XX. APPENDIX E: SENSITIVITY ANALYSIS PROGRAM

The coding shown here is for the computer program that determines, 1) the expected error, 2) the calculated variance, and 3) the actual variance of the state estimates when modeling errors are present. For the IBM 360/65, the program requires 236 K bytes of main core memory when compiled in Fortran H.

```
C SENSITIVITY ANALYSIS PROGRAM
C
      REAL*8 ELEM(1700),TEMP(800),SRAR(175),GAIN(5500),AC(175),CC(175),
     1 AF(800),AX(120),AXO(120),CXO(120),TTEM(800),CIR(175),CF(800),
     1 PRO(5500)
      REAL*8 EMAT,TMAT,ERR,TSS,SRH,TGAIN,SUB,RHS(120),EWORK(120),
     1 FWORK(120),AEWORK,GWORK,TDIAG,DIAG(120),AFWORK,DIAGK,RHSK,
     1 RHSL,ADIAG,TAF,TCIR,TSRAR
      INTEGER*2 SCOL(120),ECOL(120),COL(1700),LOOK1(175),LOOK2(175),
     1 LOOK(800),SGAIN(120),EGAIN(120),COLG(5500),CEWORK(120),
     1 CFWORK(120),LAPE(120),LAPS(120),KOL(5500),
     1 LIL(120),CODE1(120),CODE2(120),NCOM1(120),NCOM2(120),COM1(850),
     1 COM2(850),FCOM(2600),SCOM(2600),SBCOM(120),EBCOM(120),LBCOM(850),
     1 CODE(800),CSTATE(850),BCOM(850)
      COMMON/ONE/RHS,EWORK,FWORK,DIAG,ELEM,TEMP,SRAR,GAIN,AC,CC,AF,
     1 PRO,
     1 AX,AXO,CXO,TTEM,CIR,CF,NBUS,NSTATE,NMEAS,NELEM,NBUSM,
     1 NLINEM,NVOLTM,NSM,SCOL,ECOL,COL,LOOK1,LOOK2,LOOK,SGAIN,
     1 EGAIN,COLG,CEWORK,CFWORK,LAPE,LAPS,KOL
      COMMON/TWO/CODE1,CODE2,NCOM1,NCOM2,COM1,COM2,FCOM,SCOM,SBCOM,
     1 EBCOM,LBCOM
C
C NBUS = NO. OF BUSES
C NSTATE = NO. OF STATES
C NMEAS = NO. OF MEASUREMENTS
C NELEM = NO. OF OFF-DIAGONAL ELEMENTS IN COEFFICIENT MATRIX THAT WILL BE
C GENERATED IN SUBROUTINE CALELM
C
      READ(5,1) NBUS,NSTATE,NMEAS,NELEM
    1 FORMAT(4(1X,I4))
      WRITE(6,2) NBUS,NSTATE,NMEAS,NELEM
    2 FORMAT('0',1X,'NBUS=',I4,2X,'NSTATE=',I4,2X,'NMEAS=',I4,2X,
     1 'NELEM=',I4)
C
C NBUSM= NO. OF REAL OR REACTIVE BUS INJECTION MEASUREMENTS
C NLINEM = NO. OF REAL OR REACTIVE LINE FLOW MEASUREMENTS
C NVOLTM = NO. OF BUS VOLTAGE MEASUREMENTS
```

```
C
      READ(5,300) NBUSM,NLINEM,NVOLTM
  300 FORMAT(3(1X,I4))
      WRITE(6,300) NBUSM,NLINEM,NVOLTM
      NSM=NSTATE-1
C
C LOOK=LIST OF STATES AS THEY APPEAR IN THE EQUATION FOR EACH MEASUREMENT
C LOOK1=LOCATION OF FIRST ELEMENT IN LOOK LIST FOR EACH MEAS.
C LOOK2=LOCATION OF LAST ELEMENT IN LOOK LIST FOR EACH MEAS.
C
      READ(5,813) (LOOK1(K),LOOK2(K),K=1,NMEAS)
  813 FORMAT(10(I3,2X,I3))
      DO 814 K=1,NMEAS
      MA=LOOK1(K)
      MB=LOOK2(K)
      READ(5,815) (LOOK(L),L=MA,MB)
  815 FORMAT(20(I3,1X))
  814 CONTINUE
C
C CODE = LIST OF MEASUREMENTS AS THEY INCLUDE EACH OF THE STATES
C CODE1 = LOCATION OF FIRST ELEMENT IN CODE LIST FOR EACH STATE
C CODE2 = LOCATION OF LAST ELEMENT IN CODE LIST FOR EACH STATE
C CSTATE: FOR EACH STATE, CSTATE LISTS THE HIGHER NUMBERED STATES HAVING CODE
C ELEMENTS IN COMMON WITH THAT STATE
C FCOM: FOR EACH STATE, FCOM LISTS THE LOCATION OF ELEMENTS IN THE CODE LIST
C THAT ARE COMMON TO THE CODE LIST ELEMENTS LISTED UNDER HIGHER NUMBERED STATES
C SCOM: SCOM LISTS THE STORAGE LOCATION OF THOSE ELEMENTS IN CODE WHOSE
C LOCATIONS WERE LISTED IN FCOM
C COM1 = LOCATION OF THE FIRST ELEMENT IN FCOM AND SCOM LISTS FOR EACH CSTATE
C COM2 = LOCATION OF THE LAST ELEMENT IN FCOM AND SCOM LISTS FOR EACH CSTATE
C NCOM1 = LOCATION OF FIRST ELEMENT IN CSTATE LIST FOR EACH STATE
C NCOM2 = LOCATION OF LAST ELEMENT IN CSTATE LIST FOR EACH STATE
C BCOM: FOR EACH STATE, BCOM LISTS THE LOWER NUMBERED STATES HAVING CODE
C ELEMENTS IN COMMON WITH THE CODE ELEMENTS FOR THAT STATE
C LBCOM LISTS THE LOCATION OF THE BCOM ELEMENTS IN THE CSTATE LIST
C SBCOM LISTS THE LOCATION OF THE FIRST ELEMENT IN THE BCOM AND LBCOM LISTS FOR
C EACH STATE
```

```
C EBCOM LISTS THE LOCATION OF THE LAST ELEMENT IN THE BCOM AND LBCOM LISTS FOR
C EACH STATE
C COL LISTS THE COLUMNS FOR THE ELEMENTS OF EACH ROW OF THE COEFFICIENT MATRIX
C THAT WILL BE GENERATED IN THIS PROGRAM
C SCOL = LOCATION OF THE FIRST ELEMENT IN COL FOR EACH STATE
C ECOL = LOCATION OF THE LAST ELEMENT IN COL FOR EACH STATE
C NELEM = NO. OF OFF-DIAGONAL ELEMENTS IN THE COEFFICIENT MATRIX THAT WILL BE
C GENERATED IN THIS PROGRAM
C
      READ(5,821) (CODE1(K),CODE2(K),NCOM1(K),NCOM2(K),SBCOM(K),
     1 EBCOM(K),K=1,NSM)
  821 FORMAT(I3,T6,I3,T11,I3,T16,I3,T21,I3,T26,I3)
      READ(5,838) CODE1(NSTATE),CODE2(NSTATE),SBCOM(NSTATE),
     1 EBCOM(NSTATE)
  838 FORMAT(I3,T6,I3,T21,I3,T26,I3)
      DO 819 K=1,NSM
      MA=CODE1(K)
      MB=CODE2(K)
      READ(5,820) (CODE(L),L=MA,MB)
  820 FORMAT(20(I3,1X))
      NA=NCOM1(K)
      NB=NCOM2(K)
      IF(NA.LT.1) GO TO 819
      READ(5,822) (COM1(L),COM2(L),CSTATE(L),L=NA,NB)
  822 FORMAT(16(I4,1X))
      DO 878 J=NA,NB
      IA=COM1(J)
      IB=COM2(J)
      READ(5,877) (FCOM(L),SCOM(L),L=IA,IB)
  877 FORMAT(20(I3,1X))
  878 CONTINUE
  819 CONTINUE
      DO 590 K=2,NSTATE
      KK=SBCOM(K)
      LL=EBCOM(K)
      IF(KK.LT.1) GO TO 590
      READ(5,591) (BCOM(L),LBCOM(L),L=KK,LL)
```

```
  591 FORMAT(16(I4,1X))
  590 CONTINUE
      JA=CCDE1(NSTATE)
      JB=CCDE2(NSTATE)
      READ(5,839) (CCDE(L),L=JA,JB)
  839 FORMAT(20(I3,1X))
      READ(5,580) NELEM
  580 FORMAT(I4)
      WRITE(6,581) NELEM
  581 FORMAT('0',1X,'NELEM=',I4)
      READ(5,582) (CCL(K),K=1,NELEM)
  582 FORMAT(20(I3,1X))
      READ(5,490) (SCOL(K),ECOL(K),K=1,NSTATE)
  490 FORMAT(15(I4,1X))
      NF=LCCK2(NMEAS)
C
C CF = JACOBIAN MATRIX FROM STATE ESTIMATOR PROGRAM WITH MODELLING ERRORS
C PRESENT
C
      READ(5,9) (CF(K),K=1,NF)
    9 FORMAT(5(2X,E14.7))
      WRITE(6,10)
   10 FORMAT('0',2X,'CF    ELEMENTS')
      WRITE(6,11) (K,  CF(K),K=1,NF)
   11 FORMAT('0',8(1X,I3,1X,E10.3))
C
C AF = JACCBIAN MATRIX FROM STATE ESTIMATOR PROGRAM WITHOUT MODELLING ERRORS
C
      READ(5,600) (AF(K),K=1,NF)
  600 FORMAT(5(2X,E14.7))
      WRITE(6,604)
  604 FORMAT('0',2X,'ACT. F, AF')
      WRITE(6,605) (AF(K),K=1,NF)
  605 FCRMAT(10(2X,E10.3))
C
C AC = CALCULATED VALUES OF MEASUREMENTS WITHOUT MODELLING ERRORS
C
```

```
      READ(5,601) (AC(K),K=1,NMEAS)
  601 FORMAT(5(2X,E14.7))
      WRITE(6,606)
  606 FORMAT('0',2X,'ACT. C, AC')
      WRITE(6,607) (AC(K),K=1,NMEAS)
  607 FORMAT(10(2X,E10.3))
C
C AXO = STATE VECTOR USED IN CALCULATING AC
C
      READ(5,602) (AXO(K),K=1,NBUS)
      N1=NBUS+1
      READ(5,602) (AXO(K),K=N1,NSTATE)
  602 FORMAT(5(2X,E14.7))
      WRITE(6,608)
  608 FORMAT('0',2X,'ACT. XO, AXO')
      WRITE(6,609) (AXO(K),K=1,NSTATE)
  609 FORMAT('0',10(2X,E10.3))
C
C CC = CALCULATED VALUES OF MEASUREMENTS WITH MODELLING ERRORS PRESENT
C
      READ(5,603) (CC(K),K=1,NMEAS)
  603 FORMAT(5(2X,E14.7))
      WRITE(6,610)
  610 FORMAT('0',2X,'CALC. MEAS., CC')
      WRITE(6,611) (CC(K),K=1,NMEAS)
  611 FORMAT('0',10(2X,E10.3))
C
C CXO = STATE VECTOR USED IN CALCULATING CC
C
      READ(5,612) (CXO(K),K=1,NBUS)
      READ(5,612) (CXO(K),K=N1,NSTATE)
  612 FORMAT(5(2X,E14.7))
      WRITE(6,613)
  613 FORMAT('0',2X,'CALC. XO, CXO')
      WRITE(6,614) (CXO(K),K=1,NSTATE)
  614 FORMAT('0',10(2X,E10.3))
C
```

```
C AX = TRUE STATE VECTOR
C
      READ(5,615) (AX(K),K=1,NBUS)
      READ(5,615) (AX(K),K=N1,NSTATE)
  615 FORMAT(5(2X,E14.7))
      WRITE(6,616)
  616 FORMAT('0',2X,'ACT. X, AX')
      WRITE(6,617) (AX(K),K=1,NSTATE)
  617 FORMAT('0',10(2X,E10.3))
C
C CIR = DIAGONAL INVERSE COVARIANCE MATRIX OF  MEASUREMENT ERRORS FOR SYSTEM
C WITH MODELLING  ERRORS
C
      READ(5,275) (CIR(K),CIR(K+NBUSM),K=1,NBUSM)
  275 FORMAT(T16,E10.3,T28,E10.3)
      M=2*NBUSM
      N=M+NLINEM
      READ(5,276) (CIR(M+K),CIR(N+K),K=1,NLINEM)
  276 FORMAT(T18,E10.3,T30,E10.3)
      N=2*(NBUSM+NLINEM)
      READ(5,277) (CIR(N+K),K=1,NVOLTM)
  277 FORMAT(T16,E10.3)
      WRITE(6,278)
  278 FORMAT('0',2X,'CAL. INCOV., CIR')
      WRITE(6,279) (CIR(K),K=1,NMEAS)
  279 FORMAT('0',10(2X,E10.3))
C
C SRAR = DIAGONAL INVERSE COVARIANCE MATRIX OF  MEASUREMENT ERRORS FOR SYSTEM
C WITH NO MODELLING ERRORS
C
      READ(5,275) (SRAR(K),SRAR(K+NBUSM),K=1,NBUSM)
      M=2*NBUSM
      N=M+NLINEM
      READ(5,276) (SRAR(M+K),SRAR(N+K),K=1,NLINEM)
      N=2*(NBUSM+NLINEM)
      READ(5,277) (SRAR(N+K),K=1,NVOLTM)
      WRITE(6,280)
```

```
  280 FORMAT('0',2X,'ACT. INCOV.,SRAR')
      WRITE(6,279) (SRAR(K),K=1,NMEAS)
      DO 281 K=1,NMEAS
      SRAR(K)=1/DSQRT(SRAR(K))
  281 CONTINUE
C
C SRAR = SQUARE ROOT OF DIAGONAL COVARIANCE MATRIX OF MEASUREMENT ERROR FOR
C SYSTEM WITH NO MODELLING ERRORS
C
      CALL CALELM
C
C DIAG = DIAGONAL TERMS OF COEFFICIENT MATRIX CALCULATED IN SUBROUTINE CALELM
C
      WRITE(6,4)
    4 FORMAT('0',2X,'DIAG. ELEMENTS')
      WRITE(6,5) (K,DIAG(K),K=1,NSTATE)
    5 FORMAT('0',8(1X,I3,1X,E10.3))
C
C ELEM = OFF-DIAG. TERMS OF COEFFICIENT MATRIX CALCULATED IN SUBROUTINE CALELM
C
      WRITE(6,7)
    7 FORMAT('0',2X,'OFF-DIAG. ELEMENTS')
      WRITE(6,8) (K,ELEM(K),K=1,NELEM)
    8 FORMAT('0',7(1X,I4,1X,E10.3))
      DO 305 K=1,NMEAS
      I=LOOK1(K)
      J=LOOK2(K)
C
C TAF = K TH ELEMENT OF THE VECTOR, AF*(AX-AXO)
C
      TAF=0.0
      DO 306 L=I,J
      TAF=TAF+AF(L)*(AX(LOOK(L))-AXO(LOOK(L)))
  306 CONTINUE
C
C AF = K TH ELEMENT OF THE VECTOR, AC-CC+AF*(AX-AXO)
C
```

```
      AF(K)=TAF+AC(K)-CC(K)
  305 CONTINUE
      DO 800 K=1,NMEAS
      I=LOCK1(K)
      J=LOCK2(K)
      TCIR=CIR(K)
      TSRAR=SRAR(K)
C
C TEMP = MATRIX PRODUCT OF CF*CIR
C TTEM = MATRIX PRODUCT OF CF*CIR*SRAR
C
      DO 801 L=I,J
      TEMP(L)=CF(L)*TCIR
      TTEM(L)=TEMP(L)*TSRAR
  801 CONTINUE
  800 CONTINUE
      CALL REDMAT
C
C REFER TO SUBROUTINE REDMAT FOR DEFINITION OF DIAG,LAPS,LAPE,KOL,PRO,GAIN,
C SGAIN,EGAIN
C RHS(K) = DIAG(K), ALL OTHER RHS=0.0 TO FIND EACH COLUMN OF INVERSE OF
C CF'*CIR*CF. EACH COLUMN IS THEN STORED IN RHS.
C
      DO 420 K=2,NSTATE
      K1=K-1
      DO 421 II=1,K1
      RHS(II)=0.0
  421 CONTINUE
      RHS(K)=DIAG(K)
      IF(K.GE.NSTATE) GO TO 201
      K1=K+1
      DO 423 IC=K1,NSTATE
      I=LAPS(IC)
      J=LAPE(IC)
      IF(I.GT.J) GO TO 202
      RHSK=0.0
      DO 422 L=I,J
```

```
      IF(KOL(L).LT.K) GO TO 422
      RHSK=RHSK-RHS(KOL(L))*PRO(L)
  422 CONTINUE
      RHS(IC)=RHSK*DIAG(IC)
      GO TO 423
  202 RHS(IC)=0.0
  423 CONTINUE
  201 NS=NSTATE-1
      DO 823 KM=1,NS
      KSTATE=NSTATE-KM
      SRH=RHS(KSTATE)
      KK=SGAIN(KSTATE)
      LL=EGAIN(KSTATE)
      IF(KK.GT.LL) GO TO 817
      DO 824 L=KK,LL
      SRH=SRH-GAIN(L)*RHS(COLG(L))
  824 CONTINUE
  817 RHS(KSTATE)=SRH
  823 CONTINUE
C
C RHS(K)= K TH DIAGONAL TERM OF THE INVERSE OF (CF'*CIR*CF), WHICH IS THE
C CALCULATED VARIANCE OF THE K TH STATE ESTIMATE.
C
      WRITE(6,71) K,RHS(K)
   71 FORMAT('0',2X,'STATE=',I3,2X,'CAL. VAR=',E14.7)
C
C ERR = EXPECTED ERROR OF THE K TH STATE ESTIMATE
C TSS = ACTUAL VARIANCE OF THE K TH STATE ESTIMATE
C EMAT = ROW K, COLUMN L OF INV(CF'*CIR*CF)*CF'*CIR
C TMAT = ROW K, COLUMN L OF INV(CF'*CIR*CF)*CF'*CIR*SRAR
C TSS FOR STATE K IS FOUND BY SUMMING THE SQUARES OF ROW K OF
C INV(CF'*CIR*CF)*CF'*CIR*SRAR
C
      ERR=0.0
      TSS=0.0
      DO 28 L=1,NMEAS
      I=LOOK1(L)
```

```
      J=LOOK2(L)
      TMAT=0.0
      EMAT=0.0
      DO 29 M=I,J
      EMAT=EMAT+TEMP(M)*RHS(LOOK(M))
      TMAT=TMAT+TTEM(M)*RHS(LOOK(M))
   29 CONTINUE
      TSS=TSS+TMAT*TMAT
      ERR=ERR+EMAT*AF(L)
   28 CONTINUE
      ERR=ERR-AX(K)+CXO(K)
      WRITE(6,105) K,ERR
  105 FORMAT('0',2X,'STATE=',I3,2X,'EXP. ERROR=',E14.7)
      WRITE(6,30) K,TSS
   30 FORMAT('0',2X,'STATE=',I3,2X,'ACT. VAR=',E14.7)
      WRITE(7,95) K,RHS(K),ERR,TSS
   95 FORMAT(I3,2X,E14.7,2X,E14.7,2X,E14.7)
  420 CONTINUE
      STOP
      END


      SUBROUTINE CALELM
C
C SUBROUTINE FOR CALCULATING ELEMENTS OF THE COEFFICIENT MATRIX, (CF'*CIR*CF)
C
      REAL*8 ELEM(1700),TEMP(800),SRAR(175),GAIN(5500),AC(175),CC(175),
     1 AF(800),AX(120),AXO(120),CXO(120),TTEM(800),CIR(175),CF(800),
     1 PRO(5500),TEMPR(850),STORE
      REAL*8 EMAT,TMAT,ERR,TSS,SRH,TGAIN,SUB,RHS(120),EWORK(120),
     1 FWORK(120),AEWORK,GWORK,TDIAG,DIAG(120),AFWORK,DIAGK,RHSK,
     1 RHSL,ADIAG,TAF,TCIR,TSRAR
      INTEGER*2 SCOL(120),ECOL(120),COL(1700),LOOK1(175),LOOK2(175),
     1 LOOK(800),SGAIN(120),EGAIN(120),COLG(5500),CEWORK(120),
     1 CFWORK(120),LAPE(120),LAPS(120),KOL(5500),
     1 LIL(120),CODE1(120),CODE2(120),NCOM1(120),NCOM2(120),COM1(850),
```

```
     1 COM2(850),FCOM(2600),SCOM(2600),SBCOM(120),EBCOM(120),LBCOM(850)
      COMMON/ONE/RHS,EWORK,FWORK,DIAG,ELEM,TEMP,SRAR,GAIN,AC,CC,AF,
     1 PRO,
     1 AX,AXO,CXO,TTEM,CIR,CF,NBUS,NSTATE,NMEAS,NELEM,NBUSM,
     1 NLINEM,NVOLTM,NSM,SCOL,ECOL,COL,LOOK1,LOOK2,LOOK,SGAIN,
     1 EGAIN,COLG,CEWORK,CFWORK,LAPE,LAPS,KOL
      COMMON/TWO/CODE1,CODE2,NCOM1,NCOM2,COM1,COM2,FCOM,SCOM,SBCOM,
     1 EBCOM,LBCOM
C
C NELMT = NO. OF OFF TERMS IN EACH OF THE JACOBIANS, AF AND CF
C
      NELMT=LOOK2(NMEAS)
      DO 800 K=1,NMEAS
      I=LOOK1(K)
      J=LOOK2(K)
      STORE=CIR(K)
C
C GAIN = MATRIX PRODUCT, CF'*CIR
C
      DO 801 L=I,J
      GAIN(L)=CF(L)*STORE
  801 CONTINUE
  800 CONTINUE
C
C LIL =      INDEX  ARRAY USED FOR TRASPOSING GAIN AND CF
C ELEM = TRANSPOSE OF CF
C TEMPR = TRANSPOSE OF GAIN
C
      DO 804 K=1,NSTATE
      LIL(K)=0
  804 CONTINUE
      DO 805 K=1,NELMT
      I=LOOK(K)
      J=CODE1(I)+LIL(I)
      ELEM(J)=CF(K)
      TEMPR(J)=GAIN(K)
      LIL(I)=LIL(I)+1
```

```
  805 CONTINUE
C
C TDIAG = DIAG(K) = K TH DIAGONAL OF COEFFICIENT MATRIX, (CF'*CIR*CF)
C
      DO 806 K=1,NSM
      I=CODE1(K)
      J=CODE2(K)
      II=NCOM1(K)
      JJ=NCOM2(K)
      TDIAG=0.0
      DO 807 L=I,J
      TDIAG=TDIAG+ELEM(L)*TEMPR(L)
  807 CONTINUE
      DIAG(K)=TDIAG
      IF(II.EQ.0) GO TO 806
C
C TGAIN = GAIN(KK) = KKTH UPPER OFF-DIAGONAL TERM OF THE COEFFICIENT MATRIX,
C (CF'*CIR*CF)
C
      DO 808 KK=II,JJ
      KOM1=COM1(KK)
      KOM2=COM2(KK)
      TGAIN=0.0
      DO 809 LL=KOM1,KOM2
      TGAIN=TGAIN+ELEM(FCOM(LL))*TEMPR(SCOM(LL))
  809 CONTINUE
      GAIN(KK)=TGAIN
  808 CONTINUE
  806 CONTINUE
      I=CODE1(NSTATE)
      J=CODE2(NSTATE)
      TDIAG=0.0
      DO 180 K=I,J
      TDIAG=TDIAG+ELEM(K)*TEMPR(K)
  180 CONTINUE
      DIAG(NSTATE)=TDIAG
      MMM=NCOM1(1)
```

```
      M =NCOM2(1)
      IF(MMM.LT.1) GO TO 898
C
C ELEM: INCLUDES ALL OFF-DIAGONAL TERMS  OF THE COEFFICIENT MATRIX, (CF'*CIR*CF)
C
      DO 550 K=MMM,M
      ELEM(K)=GAIN(K)
  550 CONTINUE
  898 CONTINUE
      DO 551 K=2,NSM
      KSBCOM=SBCOM(K)
      KEBCOM=EBCOM(K)
      IF(KSBCOM.LT.1) GO TO 553
      DO 552 L=KSBCOM,KEBCOM
      M=M+1
      ELEM(M)=GAIN(LBCOM(L))
  552 CONTINUE
  553 CONTINUE
      KNCOM1=NCOM1(K)
      KNCOM2=NCOM2(K)
      IF(KNCOM1.LT.1) GO TO 551
      DO 554 L=KNCOM1,KNCOM2
      M=M+1
      ELEM(M)=GAIN(L)
  554 CONTINUE
  551 CONTINUE
      NSBCOM=SBCOM(NSTATE)
      NEBCOM=EBCOM(NSTATE)
      IF(NSBCOM.LT.1) GO TO 555
      DO 556 K=NSBCOM,NEBCOM
      M=M+1
      ELEM(M)=GAIN(LBCOM(K))
  556 CONTINUE
  555 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE REDMAT
C
C THIS SUBROUTINE TRIANGULARIZES THE COEFFICIENT MATRIX, (CF'*CIR*CF), AND
C CALCULATES THE FIRST COLUMN OF ITS INVERSE BY BACK SUBSTITUTION
C
      REAL*8 ELEM(1700),TEMP(800),SRAR(175),GAIN(5500),AC(175),CC(175),
     1 AF(800),AX(120),AXO(120),CXO(120),TTEM(800),CIR(175),CF(800),
     1 PRO(5500)
      REAL*8 EMAT,TMAT,ERR,TSS,SRH,TGAIN,SUB,RHS(120),EWORK(120),
     1 FWORK(120),AEWORK,GWORK,TDIAG,DIAG(120),AFWORK,DIAGK,RHSK,
     1 RHSL,ADIAG,TAF,TCIR,TSRAR
      INTEGER*2 SCOL(120),ECOL(120),COL(1700),LOOK1(175),LOOK2(175),
     1 LOOK(800),SGAIN(120),EGAIN(120),COLG(5500),CEWORK(120),
     1 CFWORK(120),LAPE(120),LAPS(120),KOL(5500)
      COMMON/CNE/RHS,EWORK,FWORK,DIAG,ELEM,TEMP,SRAR,GAIN,AC,CC,AF,
     1 PRO,
     1 AX,AXC,CXO,TTEM,CIR,CF,NBUS,NSTATE,NMEAS,NELEM,NBUSM,
     1 NLINEM,NVOLTM,NSM,SCOL,ECOL,COL,LOOK1,LOOK2,LOOK,SGAIN,
     1 EGAIN,COLG,CEWORK,CFWORK,LAPE,LAPS,KOL
C
C RHS = RIGHT HAND SIDE, ALL ELEMENTS = 0.0  EXCEPT RHS(1)=1.0, IN ORDER TO FIND
C THE FIRST COLUMN OF INV(CF'*CIR*CF)
C
      DO 101 NZ=1,NSTATE
      RHS(NZ)=0.0
  101 CONTINUE
C
C GAIN = TRIANGULARIZED MATRIX, (CF'*CIR*CF)
C SGAIN = LOCATION OF THE FIRST ELEMENT IN EACH ROW OF GAIN
C EGAIN = LOCATION OF THE LAST ELEMENT IN EACH ROW OF GAIN
C COLG = COLUMN NUMBER OF EACH ELEMENT IN GAIN
C
      RHS(1)=1.0
      SGAIN(1)=1
      EGAIN(1)=ECOL(1)
      ADIAG=1/DIAG(1)
      DIAG(1)=ADIAG
```

```
      RHS(1)=RHS(1)*ADIAG
      IND=EGAIN(1)
      DO 401  K=1,IND
      GAIN(K)=ELEM(K)*ADIAG
      COLG(K)=COL(K)
  401 CONTINUE
C
C PRO = ARRAY OF ALL MULTIPLIERS USED IN THE GAUSSIAN ELIMINATION PROCESS USED
C TO TRIANGULARIZE (CF'*CIR*CF)
C LAPS(K) = LOCATION OF THE FIRST ELEMENT IN PRO THAT PERTAINS TO THE K TH ROW
C OF GAIN
C LAPE(K) = LOCATION OF THE LAST ELEMENT IN PRO THAT PERTAINS TO THE K TH ROW
C OF GAIN
C KOL = COLUMN NUMBER OF THE ELEMENT OF (CF'*CIR*CF) THAT IS ELIMINATED BY USE
C OF EACH OF THE CORRESPONDING ELEMENTS OF PRO
C LAP = INDEX USED TO STORE THE ELEMENTS OF PRO AND KOL
C
      LAPE(1)=0
      LAP=0
C
C EWORK = WORKING ROW USED TO ELIMINATE TERMS TO THE LEFT OF THE DIAGONAL IN
C EACH ROW OF ELEM
C FWORK: SIMILAR TO EWORK, ELEMENTS ARE PASSED BACK AND FORTH BETWEEN EWORK AND
C FWORK AS EACH ELEMENT ON THE LEFT IS ELIMINATED
C CEWORK = COLUMN NUMBER FOR EACH ELEMENT OF EWORK
C CFWORK = COLUMN NUMBER FOR EACH ELEMENT OF FWORK, THESE ELEMENTS ARE ALSO
C PASSED BETWEEN CEWORK AND CFWORK AS THE ELIMINATION PROGRESSES
C KJ = INDEX FOR STORING THE ELEMENTS OF EWORK AND CEWORK
C KS = INDEX FOR STORING THE ELEMENTS OF FWORK AND CFWORK
C
      DO 405  K=2,NSTATE
      KJ=0
      KSCOL=SCOL(K)
      KECOL=ECOL(K)
      DO 402  KI=KSCOL,KECOL
      KJ=KJ+1
      EWORK(KJ)=ELEM(KI)
```

```
      CEWORK(KJ)=COL(KI)
  402 CONTINUE
      K1=K-1
      RHSK=RHS(K)
      DIAGK=DIAG(K)
C
C KIK = COUNTER USED FOR GENERATING ELEMENTS OF LAPE
C
      KIK=0
      DO 406 L=1,NSTATE
      KCE1=CEWORK(1)
      IF(KCE1.GT.K) GO TO 450
      LSE=SGAIN(KCE1)
      LEE=EGAIN(KCE1)
      AEWORK=EWORK(1)
      KIK=KIK+1
      LAP=LAP+1
      PRO(LAP)=AEWORK
      KCL(LAP)=KCE1
      RHSL=RHS(KCE1)
      RHSK=RHSK-RHSL*AEWORK
C
C KT = COUNTER USED TO PREVENT DUPLICATION OF ELEMENTS WHEN TWO ROWS ARE ADDED
C KLAD = SIGNAL USED TO PREVENT DUPLICATION
C GWORK = THE MULTIPLIED ELEMENT FROM A GAIN ROW THAT IS ADDED TO EWORK OR FWORK
C KX = COUNTER USED TO PREVENT DUPLICATION
C LX: USED TO RECORD DO LOOP INDEX
C
      KT=2
      KS=0
      KLAD=0
      IF(LSE.GT.LEE) GO TO 866
      DO 403 LT=LSE,LEE
      GWORK=GAIN(LT)*AEWORK
      KCOLG=COLG(LT)
      IF(K.EQ.KCOLG) GO TO 412
      KX=1
```

```
      IF(KT.GT.KJ) GO TO 410
  732 CONTINUE
      DO 408 KB=KT,KJ
      LX=KB
      IF(CEWORK(KB).EQ.KCOLG) GO TO 409
      IF(CEWORK(KB).GT.KCOLG.AND.KLAD.LT.1) GO TO 410
      KS=KS+1
      FWORK(KS)=EWORK(KB)
      CFWORK(KS)=CEWORK(KB)
      KX=KX+1
  408 CONTINUE
      IF(KLAD.GE.1) GO TO 403
      KT=KT+KX-1
      KS=KS+1
      FWORK(KS)=-GWORK
      CFWORK(KS)=KCOLG
      GO TO 403
  409 KS=KS+1
      FWORK(KS)=EWORK(LX)-GWORK
      CFWORK(KS)=KCOLG
      KT=KT+KX
      IF(LT.GE.LEE.AND.KT.LE.KJ) GO TO 731
      GO TO 403
  410 KS=KS+1
      FWORK(KS)=-GWORK
      CFWORK(KS)=KCOLG
      KT=KT+KX-1
      IF(LT.GE.LEE.AND.KT.LE.KJ) GO TO 731
      GO TO 403
  731 KLAD=1
      GO TO 732
  412 DIAGK=DIAGK-GWORK
      IF(LT.GE.LEE.AND.KT.LE.KJ) GO TO 731
  403 CONTINUE
      GO TO 867
  866 IF(KJ.LE.1) GO TO 399
      DO 868 LY=2,KJ
```

```
      KS=KS+1
      FWORK(KS)=EWORK(LY)
      CFWORK(KS)=CEWORK(LY)
  868 CONTINUE
  867 IF(KS.LT.1) GO TO 399
      KCF1=CFWORK(1)
      IF(KCF1.GT.K) GO TO 399
      LSF=SGAIN(KCF1)
      LEF=EGAIN(KCF1)
      AFWORK=FWORK(1)
      KIK=KIK+1
      LAP=LAP+1
      PRO(LAP)=AFWORK
      KOL(LAP)=KCF1
      RHSL=RHS(KCF1)
      RHSK=RHSK-RHSL*AFWORK
      KT=2
      KJ=0
      KLAD=0
      IF(LSF.GE.LEF) GO TO 869
      DO 433 LT=LSF,LEF
      GWORK=GAIN(LT)*AFWORK
      KCOLG=COLG(LT)
      IF(K.EQ.KCOLG) GO TO 442
      KX=1
      IF(KT.GT.KS) GO TO 440
  734 CONTINUE
      DO 438 KB=KT,KS
      LX=KB
      IF(CFWORK(KB).EQ.KCOLG) GO TO 439
      IF(CFWORK(KB).GT.KCOLG.AND.KLAD.LT.1) GO TO 440
      KJ=KJ+1
      EWORK(KJ)=FWORK(KB)
      CEWORK(KJ)=CFWORK(KB)
      KX=KX+1
  438 CONTINUE
      IF(KLAD.GE.1) GO TO 433
```

```
      KT=KT+KX-1
      KJ=KJ+1
      EWORK(KJ)=-GWORK
      CEWORK(KJ)=KCOLG
      GO TO 433
  439 KJ=KJ+1
      EWORK(KJ)=FWORK(LX)-GWORK
      CEWORK(KJ)=KCOLG
      KT=KT+KX
      IF(LT.GE.LEF.AND.KT.LE.KS) GO TO 733
      GO TO 433
  440 KJ=KJ+1
      EWORK(KJ)=-GWORK
      CEWORK(KJ)=KCOLG
      KT=KT+KX-1
      IF(LT.GE.LEF.AND.KT.LE.KS) GO TO 733
      GO TO 433
  733 KLAD=1
      GO TO 734
  442 DIAGK=DIAGK-GWORK
      IF(LT.GE.LEF.AND.KT.LE.KS) GO TO 733
  433 CONTINUE
      GO TO 870
  869 IF(KS.LE.1) GO TO 450
      DO 871 LY=2,KS
      KJ=KJ+1
      EWORK(KJ)=FWORK(LY)
      CEWORK(KJ)=CFWORK(LY)
  871 CONTINUE
  870 IF(KJ.LT.1) GO TO 450
  406 CONTINUE
  399 ADIAG=1/DIAGK
      DIAG(K)=ADIAG
      LAPS(K)=LAPE(K-1)+1
      LAPE(K)=LAPE(K-1)+KIK
      RHS(K)=RHSK*ADIAG
      KEGAIN=EGAIN(K-1)
```

```
      EGAIN(K)=KEGAIN+KS
      IF(KS.LT.1) GO TO 453
      DO 451 M=1,KS
      GAIN(M+KEGAIN)=FWORK(M)*ADIAG
      COLG(M+KEGAIN)=CFWORK(M)
  451 CONTINUE
      GO TO 453
  450 ADIAG=1/DIAGK
      DIAG(K)=ADIAG
      LAPS(K)=LAPE(K-1)+1
      LAPE(K)=LAPE(K-1)+KIK
      RHS(K)=RHSK*ADIAG
      KEGAIN=EGAIN(K-1)
      EGAIN(K)=KEGAIN+KJ
      IF(KJ.LT.1) GO TO 453
      DO 452 M=1,KJ
      GAIN(M+KEGAIN)=EWORK(M)*ADIAG
      COLG(M+KEGAIN)=CEWORK(M)
  452 CONTINUE
  453 SGAIN(K)=KEGAIN+1
      WRITE(6,890) SGAIN(K),EGAIN(K)
  890 FORMAT('0',2X,I4,2X,I4)
  405 CONTINUE
      WRITE(6,300) (DIAG(K),K=1,NSTATE)
  300 FORMAT('0',10(1X,E10.3))
      KLU=EGAIN(NSTATE)
      WRITE(6,301) (COLG(K),GAIN(K),K=1,KLU)
  301 FORMAT('0',5(1X,I3,1X,E10.3))
      WRITE(6,302) (LAPS(K),LAPE(K),K=2,NSTATE)
  302 FORMAT('0',20(1X,I5))
      WRITE(6,301) (KOL(K),PRO(K),K=1,LAP)
      NS=NSTATE-1
      DO 470 K=1,NS
      KSTATE=NSTATE-K
      SRH=RHS(KSTATE)
      KK=SGAIN(KSTATE)
      LL=EGAIN(KSTATE)
```

```
      IF(KK.GT.LL) GO TO 825
      DO 471 L=KK,LL
      SRH=SRH-GAIN(L)*RHS(COLG(L))
  471 CONTINUE
C
C RHS IS USED TO STORE THE FIRST COLUMN OF INV(CF'*CIR*CF)
C ERR,TSS,TMAT,EMAT ARE DEFINED AFTER REDMAT CALL STATEMENT IN MAIN PROGRAM
C
  825 RHS(KSTATE)=SRH
  470 CONTINUE
      WRITE(6,90) RHS(1)
   90 FORMAT('0',2X,'STATE=1',2X,'CAL. VAR.=',E14.7)
      ERR=0.0
      TSS=0.0
      DO 91 L=1,NMEAS
      I=LOCK1(L)
      J=LOCK2(L)
      TMAT=0.0
      EMAT=0.0
      DO 92 M=I,J
      EMAT=EMAT+TEMP(M)*RHS(LOOK(M))
      TMAT=TMAT+TTEM(M)*RHS(LOOK(M))
   92 CONTINUE
      TSS=TSS+TMAT*TMAT
      ERR=ERR+EMAT*AF(L)
   91 CONTINUE
      ERR=ERR-AX(1)+CXO(1)
      WRITE(6,93) ERR
   93 FORMAT('0',2X,'STATE=1',2X,'EXP. ERROR=',E14.7)
      WRITE(6,94) TSS
   94 FORMAT('0',2X,'STATE=1',2X,'ACT. VAR.=',E14.7)
      K=1
      WRITE(7,95) K,RHS(K),ERR,TSS
   95 FORMAT(I3,2X,E14.7,2X,E14.7,2X,E14.7)
      RETURN
      END
```